Cavity Cooling with a Hot Cavity

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1 Introduction

Whereas from a classical point of view the emission of radiation by an oscillating atomic dipole is a local property determined by the magnitude of the dipole moment and the angular frequency of oscillation ω, quantum electrodynamics introduces the more comprehensive concept of delocalized electromagnetic modes to describe spontaneous emission or scattering. In classical electrodynamics, the ω^4-dependence of the spontaneously emitted or Rayleigh scattered power arises from the Larmor formula for the emission by an accelerated point charge. From a quantum mechanical perspective, the spontaneous emission and the scattering rate are both proportional to the density of electromagnetic modes at the emission frequency. In free space, the mode density scales as ω^2, and multiplication by the square of the matrix element for the atom light coupling, scaling as $\omega/\hbar$, and the photon energy $\hbar\omega$ then yields the classical expression. The strong ω^4-frequency dependence is responsible not only for the blue sky, but also explains why the edible laser [1], rather than the edible maser, was experimentally realized.

If we change the density of electromagnetic modes available to the atom, e.g. by placing a resonator around it, then the emission rate is modified accordingly [2–4]. In this sense spontaneous emission is as much an atomic property as that of the electromagnetic vacuum surrounding the atom. Since the density of electromagnetic modes is a non-local feature determined by boundary conditions away from the atom, it is possible to influence the atomic emission rate by manipulating spatially extended electromagnetic modes.

Conventional Doppler cooling [5], that makes use of the conservation of momentum in scattering, was such a fantastic idea that for some time even its inventors had doubts as to its feasibility. Can you imagine to simply shine light into a vapor cell and to observe atoms near -273C moving at the speed of an ant? Then how long before we can count the frequency of light, perform precision spectroscopy of anti-hydrogen, implement magnetic motors to move Bose-Einstein condensates, realize an atomic laser, or maybe build time machines?

Doppler cooling relies on the anisotropic absorption of light by a moving two-level atom, where cooling events are favored over heating events because the incident light is tuned below the atomic resonance by an amount on the order of the Doppler effect. Conventional Doppler cooling therefore requires a closed two-level system. Alternatively, utilizing the frequency variation of the electromagnetic mode density in resonators, it is possible to devise optical cooling schemes that arise from an asymmetry in emission, rather than in absorption [6–9]. The frequency-dependent modification of emission rates inside a resonator implies a corresponding change of the light-induced mechanical forces on the atom. Since under certain circumstances the sign of the dissipative force depends exclusively on the detuning between incident light and resonator, without reference to the atomic level structure [9], this cavity cooling technique holds promise for generalizing laser cooling to arbitrary light scatterers, including atoms with a complicated level structure, molecules, or even mesoscopic particles.

Several authors have studied how to use a modification of the boundary conditions for the electromagnetic field, i.e. a passive optical resonator, to laser cool atoms in various situations, either by means of spontaneous emission from a two-level system [6–8,10], or by means of classical (coherent) Rayleigh scattering [8–12]. In this paper we analyze how a resonator with an intracavity gain medium, i.e. an active cavity, can be used to significantly improve the performance of cavity cooling, resulting in a larger cooling force and a lower temperature than those attainable with passive resonators.

2 Cavity cooling with an intracavity gain medium

The basic idea of cavity Doppler cooling or cavity sideband cooling in a passive resonator is to enhance the coherent (classical) scattering of blue photons over that of red photons, thereby extracting energy and entropy from the scatterer [9,12]. In analogy to conventional Doppler cooling, the maximum cooling force is of order

$$f_{max} = \hbar k \Gamma_0', \tag{1}$$

where $\hbar k$ is the photon momentum, v the atomic velocity, and Γ_0' the on-resonance scattering rate into the cavity. Therefore the figure-of-merit is the ratio η_0 of Γ_0' and the scattering rate Γ_{sc} into free space, and is given by [12]

$$\eta_0 = \frac{\Gamma_0'}{\Gamma_{sc}} = 2E_c \frac{\Delta\Omega}{4\pi}. \tag{2}$$

η_0 is proportional to the solid angle $\Delta\Omega$ that is subtended by the simultaneously resonant cavity modes, and to the intensity enhancement factor $E_c = F/\pi$, where F is the cavity finesse. The cavity-to-free-space scattering ratio η_0 determines the magnitude of the cavity cooling force relative to that for conventional Doppler cooling at the same scattering rate.

The scattering rate into the resonator strongly depends on the detuning Δ of the emitted light relative to the cavity resonance. For not too large detuning Δ, the frequency-dependent scattering ratio $\eta_c(\Delta) = \Gamma_c'(\Delta)/\Gamma_{sc}$ is a Lorentzian,

$$\eta_c(\Delta) = \eta_0 \frac{\gamma_c^2}{\gamma_c^2 + \Delta^2} = \frac{2E_c}{1 + (\Delta/\gamma_c)^2} \frac{\Delta\Omega}{4\pi}, \tag{3}$$

where γ_c is the amplitude decay rate constant for the cold (passive) cavity. The enhancement factor is inversely proportional to the cavity linewidth, and in terms of the cavity round trip time τ is given by $E_c = (\gamma_c \tau)^{-1}$.

Cooling is achieved by tuning the cavity resonance to the blue side of the atomic emission spectrum, such that the emission of blue-detuned motional sidebands is enhanced, and the emission of red-detuned motional sidebands is suppressed. This leads to a cooling force that is proportional to the Lorentzian cavity-to-free-space scattering ratio $\eta_c(\Delta)$, and results in a final temperature that is inversely proportional to its slope. A narrower cavity linewidth therefore has the advantages of a larger cooling force, and a lower temperature of order $k_B T \approx \hbar\gamma_c/\eta_0$ for $\eta_0 < 1$ and $k_B T \approx \hbar\gamma_c$ for $\eta_0 > 1$ [9,12].

Although current 'supermirrors' with very low scattering and absorption losses can sustain large enhancement factors $E_c > 10^4$ [13], even larger values will be necessary to attain scattering ratios $\eta \gg 1$ in confocal resonators, that have the largest cooling volume [12]. In addition, the cooling performance would be improved if one were able to vary E and consequently the cavity linewidth in real time in order to maximize the velocity capture range in the beginning, and minimize the temperature at the end of the cooling. Since in active resonators the linewidth is a function of the gain and much smaller than in passive resonators, the use of active resonators for cavity cooling represents a promising alternative.

For an atom placed inside an active resonator a large and strongly frequency-dependent scattering rate can be implemented in two ways. One is to use the resonator below laser threshold as a regenerative amplifier [14] for the emitted light. In this case the bandwidth and the enhancement factor are determined by the value of the regenerative gain. An alternative is to cool inside the slave laser of an injection-locked master-slave system [14], where the injecting field is tuned close to the low-frequency boundary of the locking range. Then for the light emitted by the atom the amplification and the bandwidth are determined by the proximity of the system to the boundary of the locking range. The cooling parameters are then conveniently controlled inside a feedback loop with the locking phase angle serving as the error signal.

2.1 Cavity cooling inside a regenerative amplifier

Since the intensity enhancement factor and the cooling force in a passive resonator are inversely proportional to the cavity loss, it should be possible to improve the cooling performance by compensating part of the round-trip loss with intracavity gain. In this scenario the system must be kept below laser threshold in order not to saturate the gain. Although spontaneous-emission noise in the gain medium will prevent operation of the system arbitrarily close to threshold, a significant bandwidth reduction can still be obtained [14].

To calculate the power that an atom radiates inside a linear regenerative amplifier, we write the field emitted by the atomic dipole into the cavity mode(s) under consideration as $\mathsf{E}_{atom}(t)e^{ikx-i\omega t}$, where $\mathsf{E}_{atom}(t)$ is a slowly-varying field envelope. As long as the scattering rate into the cavity does not exceed the cavity decay rate, the envelope of the field circulating inside the cavity can be approximated by its steady-state value $\mathsf{E}_{st}(t)$, that is governed by

$$\mathsf{E}_{st}(t) = g^2 r^2 e^{2ikL}\mathsf{E}_{st}(t) + \mathsf{E}_{atom}(t). \tag{4}$$

Here $L = \frac{1}{2}c\tau$ is the resonator length, r is the field reflection coefficient for each mirror, and g the single-pass amplitude gain of the intracavity gain medium. Introducing the detuning $\Delta = \omega - \omega_c$ relative to the nearest cavity resonance frequency ω_c, the intracavity field can be simply written as

$$\mathsf{E}_{st}(t) = \frac{\mathsf{E}_{atom}(t)}{1 - g^2 r^2 e^{i\Delta\tau}}. \tag{5}$$

Below laser threshold the denominator is non-zero and we can define a width of the regenerative amplifier ('hot-cavity bandwidth') $2\gamma_h$ by setting

$$\gamma_h\tau = 1 - g^2 r^2. \tag{6}$$

Then Eq. (5) close to resonance ($\Delta\tau \ll 1$) reduces to the Lorentzian form

$$\mathsf{E}_{st} = \frac{\mathsf{E}_{atom}}{\gamma_h\tau + i\Delta\tau}. \tag{7}$$

The modified emission rate, i.e. the change in emitted power, inside the cavity can be understood as being due to the interference between the steady-state field Eq. (7) and the primary field E_{atom} of the oscillating atomic dipole. The power emitted by the atom into the resonator is proportional to $|\mathsf{E}_{st} + \mathsf{E}_{atom}|^2 - |\mathsf{E}_{st}|^2$, rather than to $|\mathsf{E}_{atom}|^2$ as in free space. Therefore the cavity-to-free-space scattering ratio is given by

$$\eta_h(\Delta) = \frac{\Gamma_h'(\Delta)}{\Gamma_{sc}} = \frac{2E_h}{1 + (\Delta/\gamma_h)^2}\frac{\Delta\Omega}{4\pi}, \tag{8}$$

where $E_h = (\gamma_h\tau)^{-1}$ is the enhancement factor for the regenerative amplifier. This result is formally identical to that for the passive cavity Eq. (3) if one replaces the width of the cold cavity $2\gamma_c = 2(1 - r^2)/\tau$ by the smaller gain bandwidth of the regenerative amplifier $2\gamma_h = 2(1 - g^2 r^2)/\tau$. The larger enhancement factor $E_h = (\gamma_h\tau)^{-1}$ can be attributed to the effective reduction of cavity round trip loss from $1 - r^2$ to $1 - g^2 r^2$ due to the intracavity gain g.

A reduction of the hot-cavity bandwidth γ_h by several orders of magnitude compared to γ_c has been observed [14]. Inside a regenerative amplifier it should therefore be possible to increase the scattering ratio η and the cavity cooling force substantially beyond values attainable even with supermirrors.

2.2 Cavity cooling inside an injection-locked laser

The above analysis is concerned with an active resonator below threshold that regeneratively amplifies the radiation emitted by the atom. Cooling is achieved by frequency-selective amplification of the higher-energy motional sidebands. The amplified field is fed back phase coherently onto the atom via the resonator, and extracts a larger scattered power from the oscillating dipole by means of constructive interference. It is interesting to ask whether the same type of frequency dependent emission enhancement is also available inside a laser. In this case the laser field itself could serve as the incident field that is being scattered in the cooling process.

Here we propose to use an injection-locked system [14] to realize frequency dependent amplification of the scattered light. The particle to be cooled is placed inside the slave resonator, where it is irradiated by the slave field, whose frequency is equal to the master frequency ω_m. If ω_m is tuned near the low-energy edge of the injection-locking region, the slave will display gain peaked at its (higher) free-running frequency ω_c whose value and bandwidth depend on the proximity of the system to the injection-locking boundary. Then light scattered by the atom on blue motional sidebands near ω_c is amplified and fed back onto the atom, which results in cooling.

The only difference between cooling inside a regenerative amplifier and inside an injection-locked laser is that in the former the round-trip gain for the radiation emitted by the atom is determined by the small-signal gain g, while in the latter the single-pass gain is saturated to a value g_s that depends on the power and detuning of the master field. To analyze cavity cooling inside an injection-locked laser we merely need to find the saturated gain g_s as a function of the injection parameters. Just as for the regenerative amplifier, the cooling is then simply characterized by a Lorentzian cavity-to-free-space scattering ratio of the form Eq. (8), with a saturated hot-cavity bandwidth γ_s defined in analogy to Eq. (6) by

$$\gamma_s \tau = 1 - g_s^2 r^2. \tag{9}$$

To calculate g_s, we assume that the master field $\mathsf{E}_m e^{ik_m x - i\omega_m t}$ with time-independent real envelope E_m is incident onto the slave laser with free-running frequency ω_c. Both slave laser mirrors have amplitude transmission and reflection coefficients q and r, respectively. The steady-state condition Eq. (4) with the atomic source field E_{atom} replaced by the master field inside the cavity $q\mathsf{E}_m$ in the frame rotating at ω_m then reads

$$\mathsf{E}_{st} = g_s^2 r^2 e^{i\Delta_m \tau} \mathsf{E}_{st} + q\mathsf{E}_m, \tag{10}$$

Here $\Delta_m = \omega_m - \omega_c$ is the detuning of the master field relative to the free-running slave laser. Writing the steady-state field as $\mathsf{E}_{st} = \mathsf{E}_0 e^{i\phi}$ with real E_0, we find the following relation between g_s and the locking angle ϕ:

$$\tan\phi = \frac{g_s^2 r^2 \sin\Delta_m\tau}{1 - g_s^2 r^2 \cos\Delta_m\tau}. \tag{11}$$

For not too large detuning $\Delta_m\tau \ll 1$ we can express the saturated gain half-width γ_s as

$$\gamma_s\tau = 1 - g_s^2 r^2 = \frac{\Delta_m\tau}{\Delta_m\tau - \tan\phi}. \tag{12}$$

As the master is tuned towards the edges of the locking range, the locking phase ϕ approaches the values $\pm\pi/2$ [14], and the saturated bandwidth γ_s is reduced towards zero, which according to Eqs. (3) or (8) leads to a large enhancement factor $E_s = (\gamma_s\tau)^{-1}$. The injection-locking width $\pm\Delta_l$ is given by [14]

$$\Delta_l = \frac{\mathsf{E}_m}{q\mathsf{E}_0}\gamma_c. \tag{13}$$

Here E_m and $q\mathsf{E}_0$ are the master and slave laser amplitude, respectively, measured outside the slave cavity, and $2\gamma_c$ is the cold-cavity width of the slave laser. In terms of the locking bandwidth Δ_l the enhancement factor for cooling inside an injection-locked laser can be approximately written as

$$E_s = (\gamma_s\tau)^{-1} \approx \frac{|\Delta_m|}{\Delta_l - |\Delta_m|}. \tag{14}$$

The cavity cooling force becomes very large as the master detuning $|\Delta_m|$ approaches Δ_l. This behavior is analogous to that of the regenerative amplifier near laser threshold, since for $|\Delta_m| \geq \Delta_l$ the slave reverts to its free-running frequency.

Compared to cooling with a regenerative amplifier, cooling inside an injection-locked laser offers the advantage that the gain is easily controlled and stabilized in a feedback loop using the locking phase as an error signal. For instance, a standard Pound-Drever [15] or Hänsch-Couillaud lock [16] can be used to stabilize the locking angle to a value very close to $\pi/2$. This should result in reliable operation of hot-cavity cooling, while ensuring a very large cooling force and low final temperatures. Furthermore, the high intensities inside a laser cavity should allow one to cool at very large detuning from atomic or molecular resonances, while maintaining a reasonably large scattering rate.

2.3 Limitations to cavity cooling with intracavity gain due to spontaneous-emission noise

Both for passive and for active resonators the product of enhancement factor E and gain half-width γ is constant and equal to the resonator free spectral range τ^{-1}. From the relation between amplification bandwidth and round-trip gain, Eq. (6), it may then appear that arbitrarily large enhancement factors can be obtained by adjusting the round-trip gain g^2r^2 to a value very close to unity. However, at very small round-trip loss the amplification of photons that are spontaneously emitted in the gain medium will prevent stable operation, either by triggering laser action and saturating the gain in the regenerative amplifier, or by reverting the slave to its free-running frequency in the injection-locked system.

The limit due to amplified spontaneous noise is easily estimated by noting that the same mechanism is responsible for the Schawlow-Townes linewidth $\Delta\omega_{ST}$ of a laser [17]. The smallest possible value for the gain bandwidth $2\gamma_h$ of the regenerative amplifier or $2\gamma_s$ of the injection-locked system is given by $\Delta\omega_{ST}$ at threshold,

$$\gamma_{h,s} \geq \frac{1}{2}\Delta\omega_{ST}. \tag{15}$$

Consequently the maximum enhancement factor for cavity cooling with gain is given by the ratio of free spectral range τ^{-1} and $\Delta\omega_{ST}$,

$$E_{s,h} = \frac{1}{\gamma_{s,h}\tau} \leq \frac{2}{\Delta\omega_{ST}\tau}. \tag{16}$$

In general, $\Delta\omega_{ST}$ is much smaller than the cold-cavity linewidth, which should result in significantly improved cooling performance compared to passive resonators.

3 Conclusion

Resonators with gain constitute a promising possibility to improve cavity cooling. The gain bandwidth and therefore the final temperature are limited by the Schawlow-Townes linewidth. Enhancement factors larger than 10^6 appear feasible, which even for small mode solid angles would result in a cavity-to-free-space scattering ratio $\eta \gg 1$. Then the cooling of new atomic species and perhaps even of selected molecules directly from a background vapor [12] may be within reach.

Considering the title of this volume, it would be quite appropriate if the temperature of arbitrary laser-cooled light scatterers were finally limited by the fundamental linewidth of lasers. An implementation of cavity cooling with gain would also represent just another step on this amazingly successful and surprising journey that began some 25 years ago with an improbable idea [5] about how to freeze particles with that strange hot and cold, amazing and beautiful state of light. If there is one thing that Prof. Ted Hänsch is teaching us again and again, and that I am deeply grateful for, then it is not so much how science can become great fun (I suspected that already when as an undergraduate I first heard him talk about light forces, after which no other field of physics could compete), but how playfulness can become great science.

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