Ion Crystal Transducer for Strong Coupling between Single Ions and Single Photons

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A new approach for the realization of a quantum interface between single photons and single ions in an ion crystal is proposed and analyzed. In our approach the coupling between a single photon and a single ion is enhanced via the collective degrees of freedom of the ion crystal. Applications including single-photon generation, a memory for a quantum repeater, and a deterministic photon-photon, photon-phonon, or photon-ion entangler are discussed.

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Realization of efficient quantum interfaces between single photons and single matter qubits is one of the most important and challenging goals in quantum information science [1]. It enables a wide variety of potential applications ranging from scalable quantum computing schemes to quantum networks and single-photon nonlinear optics [2]. Much progress has been achieved towards the realization of such quantum information implementations over the past decade [3–18]. Most of these have been based on neutral atoms, where quantum states can be stored for long times [3–15]. The realization of a quantum optical interface for isolated single ions, which are among the most promising qubit candidates [19], is still an outstanding challenge as the achievable coupling strength is typically small under realistic experimental conditions [20,21].

In this Letter, we propose a technique to collectively enhance the coupling between single photons and single ions using ion crystals. We consider a linear chain of ions inside an optical cavity [see Fig. 1(a)]. Strong coupling between a single ion and a single photon is realized by collective enhancement of the coupling of the photon with an ensemble of N ions, given as $g_0\sqrt{N}$, where g_0 is the vacuum Rabi frequency, i.e., the coupling between a single ion and the incoming photon that enters the cavity. This means that the photon state can be mapped onto a collective internal excitation of the ions in the absorption process. Subsequently, this state will be transferred to a phonon, i.e., a motional mode of the chain. Finally, the phonon state will be mapped to a single-ion state. Since the latter two couplings can in principle be accomplished with arbitrarily strong laser fields, the collective $g_0\sqrt{N}$ coupling can dramatically improve the overall fidelity of single-photon single-ion coupling.

We will show that using this mechanism it is possible to coherently transfer with high fidelity an arbitrary internal state of a single ion onto a single photon exiting the cavity, or an arbitrary state of a single incoming photon onto an internal ionic state. This can be used for quantum coupling of single-ion qubits in distant cavities or, alternatively, for nonlinear quantum operations (quantum gates) between single-photon qubits. In what follows we present an analysis taking into account the inhomogeneous spacing of the ions in a linear trap. This resulting inhomogeneous coupling between the cavity photon and the ions has a sizable effect on the fidelities, and we suggest an approach for the phonon-collective internal transition that will mimic the collective internal-photon transition. The total fidelity thus can be maximized by having the same relative coupling constants on each ion for both transitions.

First, we briefly outline our approach for achieving strong single-photon single-ion coupling in an optical cavity with intermediate coupling to a single ion. The system consists of a string of N ions with a Λ-level internal structure as shown in Fig. 1(b), i.e., with ground states $|0\rangle$ and $|1\rangle$ and an excited state $|e\rangle$. g_0 is the vacuum Rabi frequency on the $|0\rangle \to |e\rangle$ transition, and the transition $|1\rangle \to |e\rangle$ is driven by a classical field with Rabi frequency Ω_1 and homogeneous coupling to all ions. We also assume a quadrupole electric transition between the $|0\rangle$ and $|1\rangle$ states, with Rabi frequency Ω. The collective internal ion excitation consists then of the Dicke-like states $|\underline{\mathbf{0}}\rangle = |0_1 0_2 \ldots 0_N\rangle$ and $|\underline{\mathbf{1}}\rangle \propto \sum_i g_i |0_1 0_2 \ldots 1_i \ldots 0_N\rangle$, etc., which takes into account the inhomogeneous coupling of the ions to the cavity field with coupling coefficients $g_i = g_0 \sin(k z_i^0)$. Here g_i is the coupling of the cavity photon to ion i, where z_i^0 is the equilibrium position of the ion, and k the photon wave vector. The single-ion excitation is a metastable state $|1'\rangle_i$, where $|1'\rangle$ can be the same or different from $|1\rangle$. In our approach, in step I we map the probe photon onto the collective ion excitation, $|\underline{\mathbf{0}}\rangle \to |\underline{\mathbf{1}}\rangle$, via a stimulated Raman adiabatic passage (STIRAP) process [22]. The probe transition is thus collectively enhanced by $g_0\sqrt{N}$ [12]. Step II consists of coupling this state to a phonon $|\underline{\mathbf{1}}; n_{\text{pn}} = 0\rangle \to |\underline{\mathbf{0}}; n_{\text{pn}} = 1\rangle$ [Fig. 1(c)], where n_{pn}

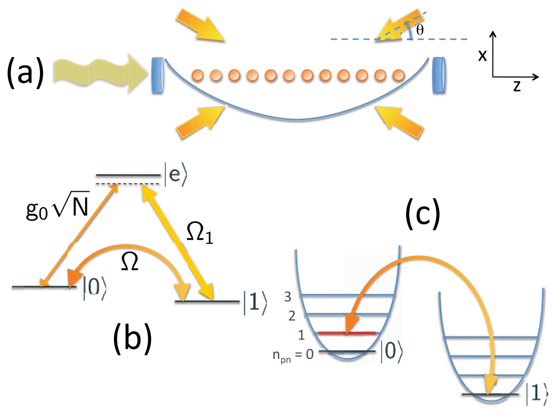

FIG. 1 (color online). Schematic of the setup. (a) Chain of N ions in cavity with incoming photon and two pairs of off-axis lasers comprising a quadrupole field. (b) Ionic level scheme. States $|0\rangle$ and $|1\rangle$ are metastable states (e.g., $3D_{5/2}$ and $4S_{1/2}$ in $^{40}\mathrm{Ca}^+$), and $|e\rangle$ is an excited state (e.g., the $4P_{3/2}$ state in the same ion). For step I, the transition $|0\rangle \to |e\rangle$ is coupled by a single photon with the effective Rabi frequency $g_0\sqrt{N}$. The Ω_1 laser is directed perpendicularly to the chain. For step II, the states $|0\rangle$ and $|1\rangle$ are coupled by a quadrupole transition. (c) Coupling to a phonon mode. The harmonic energy level of the phonon excitations, e.g., of the c.m. mode, is denoted for different internal states. The singly excited phonon mode can be addressed by a transition frequency that is red detuned from the bare ionic transitions by the frequency of the phonons ω_{pn} by a quadrupole transition. The laser scheme for this part is depicted in (a).

denotes the number of phonons in a selected mode, e.g., axial center of mass (c.m.), via an adiabatic passage process, by using the quadrupole transition coupling $|0\rangle$ and $|1\rangle$. In step III, the phonon is mapped onto a single ion j, i.e., $|\underline{\mathbf{0}}; n_{pn} = 1\rangle \to |0_1 \ldots 1_j \ldots 0_N; n_{pn} = 0\rangle$. The main limitation to this scheme is the compromise between large N for enhancing the $g_0\sqrt{N}$ coupling strength of the first step, and not too large N such that the coupling to the phonons, scaling inversely with the mass of the system, is not reduced. We find about 40 ions to be both experimentally feasible and providing sufficient coupling on both transitions. Thus single-photon single-ion coupling is achieved.

The details of the scheme are most easily understood by describing the time reversed process, so we start by analyzing how to map a single-ion state onto a phonon state (step III).

In order to map the state of the ith ion, we propose to use a Raman transition tuned to the red sideband, see Figs. 1(b) and 1(c), from $|0_1 0_2 \ldots 1_i \ldots 0_N; n_{pn} = 0\rangle$ to $|\underline{\mathbf{0}}; n_{pn} = 1\rangle$. This process can be achieved with fidelity $F_0 = |\langle \psi_f | \psi(t)\rangle|^2$ larger than 0.99, where $|\psi_f\rangle$ and $|\psi(t)\rangle$ are the ideal final state and the real state of the system at time t, respectively, when considering the full Hamiltonian without rotating-wave approximation, and the effect of the other passive phonon modes in the trap (see below). The ion-to-phonon transfer is standard in trapped ion technology [19] and can be done with large fidelity: The carrier Stark shift can be canceled by detuning the laser, and the effect of the other modes is negligible for this Rabi frequency. Individual ion addressing is easy in this regime.

For step II, in order to couple the phonon to a collective internal ion excitation $|\underline{\mathbf{0}}; n_{pn} = 1\rangle \to |\underline{\mathbf{1}}; n_{pn} = 0\rangle$, we assume that states $|1\rangle$ and $|0\rangle$ are connected by a metastable quadrupole electric transition. The reason for considering this kind of transition here is that, as opposed to step III above, there is a phase matching condition: the collective excitation has to fit the standing wave pattern of the final photon in the cavity for optimal coupling, due to the ionic inhomogeneous spacing. At first glance, a simple Raman transition would be the easiest choice here. However, because of the need to align the coupling laser(s) optimally with the modes, we find a quadrupole transition made from two pairs of lasers works best [depicted in Fig. 1(a)]. Thus we consider a slightly noncoaxial standing laser field configuration, with two pairs of lasers pairwise opposite in the x direction, with one pair pointing rightwards and the other one pointing leftwards in the z direction; see Fig. 1(a). The four lasers will give a joint field of

$$A(x, z, t) \propto \cos(k_x x)\sin(kz)\cos(\omega_0 t), \tag{1}$$

i.e., for all the ions with $x = 0$, this gives $\sin(kz)\cos(\omega_0 t)$ with a correction in x fluctuations that is quadratic in the Lamb-Dicke parameter, and thus can be neglected. These lasers will have frequency ω_0, resonant with the quadrupolar transition. The quadrupolar Hamiltonian contains the gradient of the field, such that the resulting Hamiltonian will have a $\cos(kz)$ dependence and will exhibit the typical standing wave sinusoidal pattern needed, matching the cavity mode: $\cos(kz_i) \simeq \cos(kz_i^0) - k\delta z_i \sin(kz_i^0)$, where δz_i is the fluctuation of the position operator z_i around the equilibrium position, z_i^0. This way, the ions will couple to the phonon with equal relative couplings as to the photon, thus maximizing the fidelity. Also, due to the angle θ the lasers make with the cavity axis, their frequency will correspond to the quadrupole electric transition, which usually is different from the cavity frequency. The resulting complete Hamiltonian reads

$$\begin{aligned} H_1 = {}& \omega b_{pn}^\dagger b_{pn} + \sqrt{3}\omega \tilde{b}_{pn}^\dagger \tilde{b}_{pn} + \Omega \sum_i (\sigma_i^+ + \sigma_i^-) \\ & \times \Big\{ \cos(kz_i^0) - \sin(kz_i^0) \Big[\frac{\eta}{\sqrt{N}} (b_{pn} + b_{pn}^\dagger) \\ & + \frac{\tilde{\eta}}{\sqrt{N}} (\tilde{b}_{pn} + \tilde{b}_{pn}^\dagger) \Big] \Big\} + \Delta \sum_i |1\rangle_i\langle 1|, \end{aligned} \tag{2}$$

where $b_{pn}^\dagger$ is the main phonon mode, e.g., the c.m. mode, that we will use for the protocol, and η the single-ion Lamb-Dicke parameter. In addition, the effect of the other modes will be summed up in $\tilde{b}_{pn}^\dagger$, whose frequency we take as $\sqrt{3}\omega$ (the stretch mode frequency, nearest to the c.m. one), by considering a larger Lamb-Dicke parameter for this mode. Our estimates indicate that choosing $\tilde{\eta} = 0.4 = 4\eta$ is conservative enough to slightly overestimate the spurious effect of the other modes. Accordingly, this is the value we will consider. The other quantities are $\Omega_{\max} = 0.01\omega$, where $\Omega(t)$ has a Gaussian profile, and $\eta = 0.1$. This operation is optimal if one performs an adiabatic sweep of the detuning Δ over the resonance with respect to the red sideband of the $|0\rangle \rightarrow |1\rangle$ transition as plotted in Fig. 1(c). We take a maximum value of the detuning of $|\Delta - \omega| = 8 \times 10^{-3}\omega$, and we consider a chirp with linear dependence on time. The process fidelity is given by

$$F_1 = \int d\alpha d\beta \langle \psi_f | \mathrm{Tr}[U|\psi_i\rangle\langle\psi_i|U^\dagger]|\psi_f\rangle \times \delta(1 - |\alpha|^2 - |\beta|^2),$$

where $|\psi_i\rangle = \alpha|\underline{\mathbf{0}}; n_{\rm pn} = 0\rangle + \beta|\underline{\mathbf{0}}; n_{\rm pn} = 1\rangle$ and $|\psi_f\rangle = \alpha|\underline{\mathbf{0}}; n_{\rm pn} = 0\rangle + \beta|\underline{\mathbf{1}}; n_{\rm pn} = 0\rangle$, U is the evolution operator associated with H_1, and the trace is taken over the spurious mode. F_1 is depicted in Fig. 2(a) as a function of the ion number N. While the main phonon mode, the c.m. mode, should be cooled down to the ground state for performing the protocol, the remaining modes in principle may be cooled just to the Doppler limit. We verified that there is no transfer of population from states $|\underline{\mathbf{0}}; n_{\rm pn} = 1\rangle$ and $|\underline{\mathbf{1}}; n_{\rm pn} = 0\rangle$ to other spurious phonon modes, given that they are far off-resonant. In addition, the Stark shifts induced by the spurious modes are negligible at the Doppler limit. Accordingly, these modes need not be cooled down to the ground state. For the matching to the cavity mode to work, the quadrupolar interference pattern needs to be interferometrically stable with respect to the cavity standing wave.

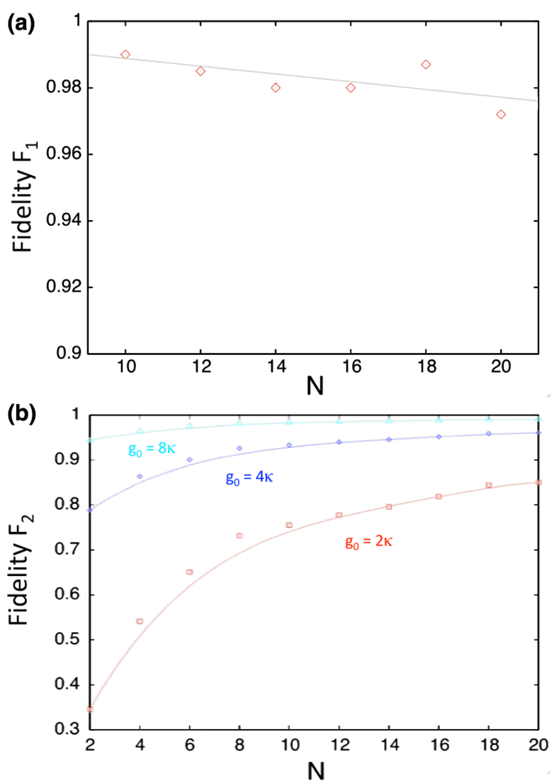

FIG. 2 (color online). (a) Process fidelity F_1 for the transfer of one phonon onto one collective excitation versus the number of ions N. The parameters used are $\Omega_{\max} = 10^{-2}\omega$, $\eta = 0.1$, $\tilde{\eta} = 0.4$. The line is an average to guide the eye. (b) Fidelity F_2 for the transfer of one collective excitation to one photon versus the number of ions N. The parameters used are $\Omega_1 = 50\kappa$, $\Gamma = 10\kappa$, $\Delta = 0$. The lines are averages to guide the eye.

Finally, step I consists of coupling the collective ion excitation to the cavity mode $a^\dagger$, that subsequently will exit the cavity [23]. This can be done using a straightforward Raman transition in a STIRAP setup, between levels $|\underline{\mathbf{1}}; n_{\rm ph} = 0\rangle$ and $|\underline{\mathbf{0}}; n_{\rm ph} = 1\rangle$ [see Fig. 1(b)]. The Ω_1 laser is directed perpendicularly to the chain, such that no additional phases are introduced. The Hamiltonian and master equation for this system read

$$H_2 = \sum_i [\Omega_1(|e\rangle_i\langle 1| + |1\rangle_i\langle e|) + g_0 \sin(k z_i^0) \times (|e\rangle_i\langle 0|a + \mathrm{H.c.})] + \Delta \sum_i |e\rangle_i\langle e|, \quad (3)$$

$$\dot{\rho} = -i[H_2, \rho] + \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \Gamma \sum_i (|1\rangle_i\langle e|\rho|e\rangle_i\langle 1| + |0\rangle_i\langle e|\rho|e\rangle_i\langle 0| - |e\rangle_i\langle e|\rho - \rho|e\rangle_i\langle e|), \quad (4)$$

with cavity decay rate κ. We can describe the transition using a STIRAP [22] pulse. The fidelity F_2 of this process will be dominated by spontaneous emission from $|e\rangle$

$$F_2 = 1 - 2\Gamma \int_0^\infty dt \sum_i \langle e_i|\rho(t)|e_i\rangle. \quad (5)$$

We plot F_2 in Fig. 2(b) versus the number of ions N. Note that F_2 increases towards 1 with the number of ions N. This growth is especially significant for low N. For $\omega/2\pi = 1$ MHz, the total transfer time of steps I–III is of about 2.3 ms for 20 ions. For the joint process fidelity of steps I–III, we obtain an optimal value of 0.98 for $N = 18$ and $g_0 = 8\kappa$.

Having explained the basic setup, we can now discuss some potential extensions and applications. One such extension, two or more excitations, can be done in a straightforward manner. In this case, we consider a superposition state with n photons $\alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_n|n\rangle$ which is mapped to the internal state $\alpha_0|0_1 0_2 \ldots 0_N\rangle + \alpha_1|0_1 \ldots 1_{i_1} \ldots 0_N\rangle + \cdots + \alpha_n|0_1 \ldots 1_{i_1} \ldots 1_{i_n} \ldots 0_N\rangle$, where i_1 through i_n denote n definite (potentially neighboring) ions. The mapping process can then be achieved much the same way as described above, but the "single" ion-to-phonon step needs n ions to be addressed individually by one or more lasers. In the phonon stage, $|n\rangle$ corresponds to $n_{\rm pn} = n$, in the collective-ion-excitation stage to the Dicke state with n excitations. The collective enhancement factor increases in this case to $\binom{N}{n}^{1/2}$. For the coherent transfer from two excitations to two photons, for example, we obtain an F_2 (which is a lower bound to the probability of no spontaneous emission in the two-excitation case) from Eq. (5) of 0.97 for 12 ions, with the parameters $\Omega_1 = 50\kappa$, $\Delta = 0$, $\Gamma = 10\kappa$, $g_0 = 8\kappa$.

An important practical consideration for the implementation of any such operations concerns mapping of incident photon states into the ion crystal. In the ideal limit, perfect mapping can be achieved, via time reversal of the spin-photon mapping procedure discussed above. In practice, the fidelity of this process will be limited by finite optical depth, i.e., by the number of ions in the crystal. The procedure of coupling photons into ensembles without and with cavities has been investigated in detail previously [17,18,24]. In general, for optimally chosen coupling strategies, the storage efficiencies will be similar to the retrieval efficiencies [14,15].

There are multiple options for this scheme to be used as a photonic gate, i.e., a nonlinearity that acts on two or more photons. In the general case we will consider a superposition $|\psi_{\rm ph}\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$ of 0, 1, and 2 photons for the incoming state, and the aim will be to introduce a minus sign in the two-photon state, to get $U^{(2)}|\psi_{\rm ph}\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle - \alpha_2|2\rangle$. A way to achieve that is based on transfer of the state $|\psi_{\rm ph}\rangle$ to a superposition of 0, 1, and 2 collective excitations, $|\psi_{\rm col}\rangle = \alpha_0|\underline{\mathbf{0}}\rangle + \alpha_1|\underline{\mathbf{1}}\rangle + \alpha_2|\underline{\mathbf{2}}\rangle$, where $|\underline{\mathbf{2}}\rangle$ is the collective Dicke-like state with two excitations, $|\underline{\mathbf{2}}\rangle \propto \sum_{i\neq j} g_i g_j |0_1 0_2 \ldots 1_i \ldots 1_j \ldots 0_N\rangle$. Transferring subsequently to the superposition of zero-, one-, and two-phonon states, $|\psi_{\rm pn}\rangle = \alpha_0|n_{\rm pn} = 0\rangle + \alpha_1|n_{\rm pn} = 1\rangle + \alpha_2|n_{\rm pn} = 2\rangle$, and from it to the same superposition of single-ion states, one would get $\alpha_0|0_1 0_2 \ldots 0_N\rangle + \alpha_1|0_1 \ldots 1_{i_1} \ldots 0_N\rangle + \alpha_2|0_1 \ldots 1_{i_1} 1_{i_1+1} \ldots 0_N\rangle$. We propose then to perform a standard 2-qubit phase gate upon the internal states of ions i_1 and $i_1 + 1$, introducing a minus sign upon the $\alpha_2|0_1 \ldots 1_{i_1} 1_{i_1+1} \ldots 0_N\rangle$ state. For example, the quantum-bus [25] or Sørensen-Mølmer [26] gates could be implemented in a straightforward manner. Subsequently, one would undo all the previous steps, and retrieve the photonic state with the minus sign incorporated upon the $\alpha_2|n_{\rm ph} = 2\rangle$ component of the state, thus completing the two-photon gate. A further possibility could be to consider the change in absorption of the cavity produced by the presence or absence of an excitation in the ions inside the cavity [27]. Thus, the phase of an incoming photon would be conditionally modified in the case where a former photon was previously absorbed.

From the above descriptions it is obvious that there is a large number of applications possible with this scheme. This includes quantum gates and quantum nondemolition measurements of single-photon qubits [2], efficient entanglement purification in ion-based quantum repeater schemes [28], efficiency enhancement in probabilistic ion entanglement schemes [29], as well as deterministic quantum gates between distant trapped ions [23].

Finally, we point out that the here envisioned level scheme may be attained in several presently possible ion trap setups such as $^{40}Ca^+$ or $^{88}Sr^+$ trapped ions.

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