Multislip Friction with a Single Ion - Supplemental Material

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In this supplement, we detail an iterative procedure to calculate slip probabilities p_i from the experimentally measured fluorescence traces of the ion, with peak heights f_i. We begin with a basic model, and then add measured corrections for imperfect experimental conditions. We include a brief discussion of the measurement of the fluorescence peaks themselves. Finally, we conclude with a comment on the calculation of the frictional energy dissipation shown in Fig. 4b of the main text. Other details of the apparatus are discussed in Refs. [1–4].

EXTRACTING LOCALIZATION PROBABILITIES - BASIC MODEL

We consider a multistable potential landscape with local minima labeled $\alpha, \beta, \gamma, ...$ from left-to right, with an ion localized in the left-most minimum α. As described in the main text, the landscape is the sum of a sinusoidal periodic potential and a harmonic trap potential. Translating the harmonic trap to the right tilts the overall potential, causing minimum α to eventually become an inflection point, and the second-left-most minimum β to become the new left-most minimum. The appearance of the inflection point marks a slipping point: the ion is forced to slip, cool, and re-localize in the various remaining minima with some probability. We make no assumption about the nature of this probability distribution.

We can determine the probability distribution directly from ion fluorescence traces. An ion's fluoresence peaks at a slipping point. The height of the peak, averaged over many realizations of the experiment, is determined by the fraction of slip events occuring at that time; the more often an ion slips, the higher the corresponding peak. The first peak, with height f_1, is always the largest because the ion is always initially prepared in the same global minimum. We normalize all populations to f_1, which represents the full ion ensemble population; consequently, f_1 acts as a conversion between fluorescence and population. After this first slip, the ion population localizes among the various empty minima: the new left-most minimum β has population p_A, where p_A denotes the probability of localizing into the left-most minimum. The next-left-most minimum γ has population p_B, where p_B is the probability of localizing into the next-left-most minimum, etc.

We continue to translate the harmonic trap as before. After a translation of one lattice spacing a, the left-most minimum β, with ion population p_A, becomes an inflection point. Its ion population slips, producing peak 2 with height $f_2 = f_1 \cdot p_A$. The population of the remaining minima increase: the new left-most minimum γ, for example, now has population $f_2/f_1 \cdot p_A + p_B$, which will eventually slip and form the third fluorescence peak $f_3 = f_2 \cdot p_A + f_1 \cdot p_B$ [5].

Iterating this procedure over several slip events gives us a general relation for the peak heights:

$$f_i = f_{i-1} \cdot p_A + f_{i-2} \cdot p_B + f_{i-3} \cdot p_C + \cdots , \qquad (1)$$

where $f_{i-j} = 0$ if $i - j \leq 0$. Thus, we can solve a set of coupled algebraic expressions to obtain the slip probability distribution $\{p_A, p_B, p_C\}$ from the measured height distribution $\{f_i\}$:

$$f_2 = f_1 \cdot p_A, \qquad (2)$$

$$f_3 = f_2 \cdot p_A + f_1 \cdot p_B, \qquad (3)$$

$$f_4 = f_3 \cdot p_A + f_2 \cdot p_B + f_1 \cdot p_C. \qquad (4)$$

Note that we are perfectly constrained; there is no need to add the constraint $p_A + p_B + p_C = 1$. This condition, however, can be used to verify errors in our extraction of proabilities arising, for example, from thermal hopping. Solving this set of Eqs. 2-4 results in Eq. 2 of the main text.

CORRECTIONS TO THE BASIC MODEL

Eq. 1 is a good working model for extracting slip probabilities, but it is only valid for perfect experimental conditions. The following are two corrections that we make to our basic model to account for experimental imperfections.

Misinitialization correction

The basic model assumes that 100% of the ion ensemble population is initially localized in minimum α and slips together to form the first fluorescence peak f_1. In the experiment, we initially localize the ion in the global minimum (at high η, this is the central of multiple minima), and a small fraction of the total ion population often spills over equally into the two minima adjacent to the global minimum. The small misinitialization in the left-adjacent minimum, in particular, can be observed as a tiny fluorescence bump that precedes the first large fluorescence peak. By measuring the bump fluorescence f_{bump}, we can determine the fraction of the total ensemble that was misinitialized and account for its contribution to the adjacent minimum.

The misinitialization correction changes the slip probability distribution by less than 10%.

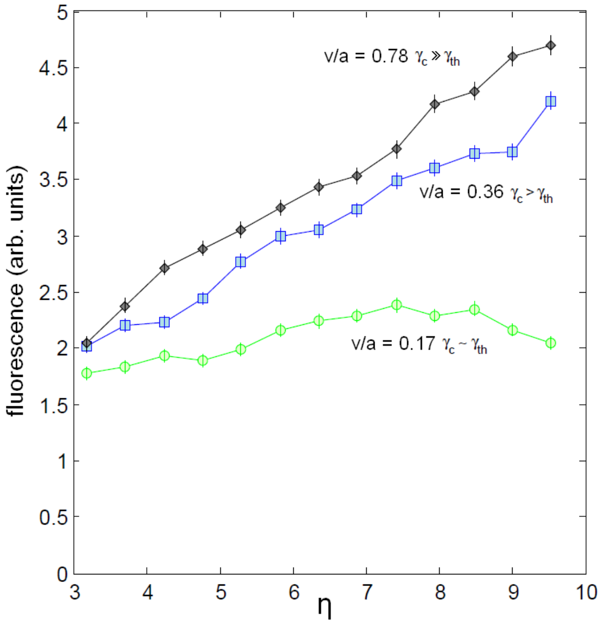

FIG. 1: First peak fluorescence f_1 across a range of values of the corrugation parameter η and trap translation speed v. The fastest curve (black diamonds), which is least subject to thermal hopping, is used as a reference. Note the increase of fluorescence with η, due to the dependence of η on optical-lattice intensity. Any deviation from a monotonic increase with η is due to thermal hopping.

Thermal hopping correction

The basic model also neglects thermal hopping. An ion, due to its finite temperature, has a non-negligible probability of being thermally excited and localizing in another minimum, without producing an (averaged) fluorescence peak at a well-defined slip time. Such an ion will not contribute to the fluorescence peak that forms at the true slipping point and thus will lead to the assignment of an incorrect localization probability. To determine the fraction of ions that prematurely hops, we compare the first peak fluorescence f_1 for a given η across different velocity datasets (Fig. 1). The dataset with the fastest translation ($v/a = 0.78\ \gamma_c \gg \gamma_{th}$) can be used as a reference, as it is in a regime in which thermal hopping is negligible [3]. With this dataset as a reference, we determine the fraction of the ion population that hops without contributing to the fluorescence peak, and we can account for this population's contribution to the adjacent minimum.

The thermal hopping correction changes the slip probability distribution by less than 20% for the slowest translation speed $v/a = 0.17\gamma_c$ and less than 10% for the middle translation speed $v/a = 0.36\gamma_c$. We note that while we can correct for thermal hopping out of the left-most minimum just prior to a slip (as discussed above), we do not have a way of rigorously correcting for thermal hopping amongst the other minima, in between slips. It is this latter effect that is responsible for deviation from the model in Fig. 3a of the main text.

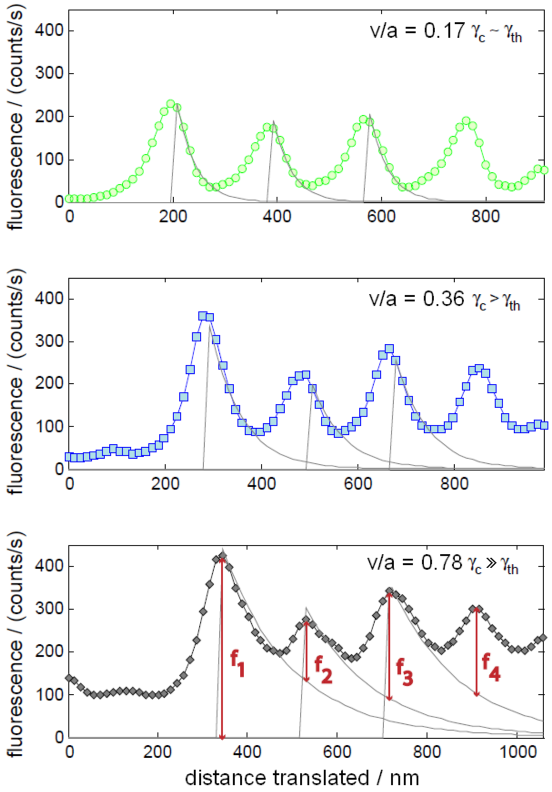

FIG. 2: Fluorescence traces, and exponential fits, for $\eta = 7.95$ across three different velocities, as a function of the harmonic trap translation. At the slowest velocity, thermal hopping is pronounced, leading to smoothing of any difference in peak heights. At the highest velocity, recooling causes a pronounced exponential fluorescence decay that must be subtracted. For the middle velocity, neither thermal hopping nor recooling effects are especially pronounced.

CORRECTION OF FLUORESCENCE PEAK HEIGHTS DUE TO FINITE COOLING RATE

Finally, we note that measuring the heights of the fluorescence peaks is complicated by the finite recooling rate γ_c of the system. Fluorescence from a slip may still

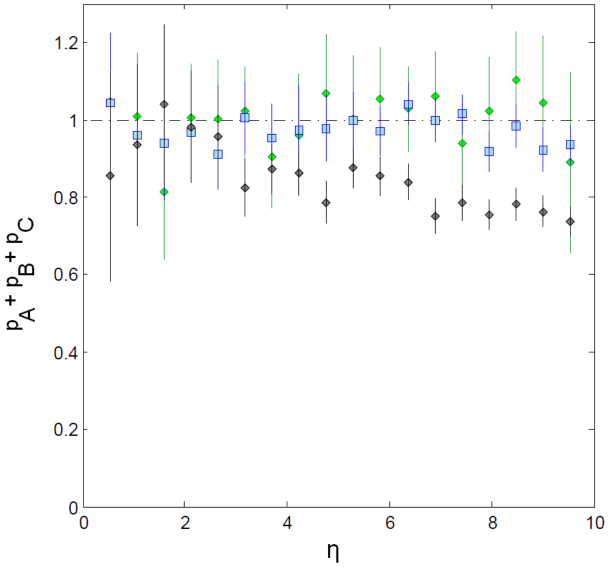

FIG. 3: Summed probabilities as a function of the corrugation parameter η across three different transport velocities ($v/a = 0.17\ \gamma_c$ for green circles, $v/a = 0.36\ \gamma_c$ for blue squares, $v/a = 0.78\ \gamma_c$ for black diamonds). The fastest dataset requires normalization.

be exponentially decaying at the time of the following slip. If one does not take this effect into account, the following slip's peak height will be overestimated. This effect is especially pronounced at high velocity, where v/a approaches γ_c. Thus, we fit each peak's fluorescence to a decaying exponential and subtract the value of this exponential when measuring peak heights (see Fig. 2). We note that this method is imperfect, as seen in Fig. 3, where the highest velocity dataset shows that $p_A + p_B + p_C < 1$. To correct for this, we re-normalize all datasets to ensure $p_A + p_B + p_C = 1$. This correction changes the probability distribution by less than 5% for the two slower velocity datasets, but amounts to a less than 20% correction for the fastest dataset $v/a = 0.78\gamma_c$.

We also see effects from the finite thermal hopping rate γ_{th}, which tends to smooth out any difference in peak heights. This effect is most pronounced at the lowest velocity and is partially corrected, as discussed above.

FRICTIONAL ENERGY DISSIPATION

The frictional energy U_{diss} dissipated by our laser cooling system after each slip event can be determined from detailed knowledge of the energy landscape:

$$\begin{aligned} U_{diss} = p_A\Big(V_0 - V_A(\tau)\Big) \\ + \frac{p_B}{2}\Big(V_0 - V_B(\tau)\Big) \\ + \frac{p_C}{3}\Big(V_0 - V_C(\tau)\Big). \end{aligned} \tag{5}$$

Here, V_0 is the energy of the ion at the slipping time, and $V_i(\tau)$ is the energy of the ith minimum at the localization time τ. These energy values are calculated from the Prandtl-Tomlinson model (Eq. 1 of the main text).

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[4] A. Bylinskii, D. Gangloff, I. Counts, V. Vuletić. Nature Materials **15**, 717-721 (2016).
[5] Note that we assume the ions experience the same slip probabilities $\{p_A, p_B, p_C\}$ at each successive slip event.