

Optomechanical Cavity Cooling of an Atomic Ensemble: Supplemental Material

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In this supplement, we begin in Sec. I by reviewing the description—previously derived by Murch *et al.* in Ref. [1]—of a trapped atomic ensemble as a single-mode mechanical oscillator coupled to the cavity. In Secs. II A-II D, we calculate the spectrum of intensity fluctuations at the output of the symmetric optical cavity, and we relate this spectrum to the spectrum of position fluctuations of the mechanical oscillator itself. We closely follow the approach of Marquardt *et al.* [2] for calculating the displacement spectrum of the mechanical oscillator, and we additionally include lowest-order effects of laser phase noise [3]. After accounting in Sec. II E for technical effects in our photodetection of the cavity transmission, we derive in Sec. II F the relation of measured photocurrent fluctuations to the occupation $\langle n \rangle$ of the collective mode. Our calibration of atom number is described in Sec. III. Finally, in Sec. IV we describe a measurement of the thermodynamic temperature associated with all N modes of the ensemble’s axial motion.

I. HAMILTONIAN AND COLLECTIVE MODE

We first consider the Hamiltonian H_{sys} of an ensemble of N atoms that are harmonically trapped, with identical trap frequencies ω_t , at various positions ξ_i along the cavity axis. For dispersive coupling of the atoms to a cavity mode (“probe” mode) of frequency ω_0 with annihilation operator \hat{a} , we have

$$H_{\text{sys}} = \sum_{i=1}^N \left[\frac{1}{2} m \omega_t^2 (\hat{x}_i - \xi_i)^2 + \frac{\hat{p}_i^2}{2m} + \hbar \Omega \sin^2(k \hat{x}_i) \hat{a}^\dagger \hat{a} \right] + \hbar \omega_0 \hat{a}^\dagger \hat{a}. \quad (1)$$

Here, the atom-probe interaction is quantified by $\Omega = g^2/\Delta$, in terms of the vacuum Rabi frequency $2g$ and detuning $\Delta \gg \Gamma$ of the probe mode from the atomic resonance with linewidth Γ . In the Lamb-Dicke regime, where the motion $\tilde{x}_i \equiv \hat{x}_i - \xi_i \ll k^{-1}$ of each atom in its trap is small compared to the probe wavelength, the optomechanical interaction term can be expressed in terms of a single position variable $\hat{X} \equiv N^{-1} \sum_{i=1}^N \sin(2k\xi_i) \tilde{x}_i$ corresponding to the collective mode \mathcal{C} discussed in the main text. The momentum conjugate to \hat{X} is expressed in terms of the single-atom momenta \hat{p}_i as $\hat{P} = N \sum_{i=1}^N \sin(2k\xi_i) \hat{p}_i / \sum_{i=1}^N \sin^2(2k\xi_i)$, such that \hat{X} and \hat{P} obey the canonical commutation relation $[\hat{X}, \hat{P}] = i\hbar$. To describe the full motion of the N -atom ensemble, one can construct an orthogonal basis comprising \hat{X} , \hat{P} , and additional coordinate pairs representing $3N - 1$ other modes of ensemble motion with energy H_\perp ; in this basis, the system Hamiltonian becomes

$$H_{\text{sys}} = \frac{1}{2} M \omega_t^2 \hat{X}^2 + \frac{\hat{P}^2}{2M} + \hbar \left(\omega_0 + \delta\omega_N + \mathcal{G} \hat{X} \right) \hat{a}^\dagger \hat{a} + H_\perp, \quad (2)$$

where $M = mN^2 / \sum_{i=1}^N \sin^2(2k\xi_i)$ represents the effective mass of the collective mode, $\delta\omega_N = \Omega \sum_{i=1}^N \sin^2(k\xi_i)$ is an overall shift of the cavity resonance due to the atoms, and $\mathcal{G} = N\Omega k$ is the change in this cavity shift per unit displacement of the collective mode.

In Eq. 2, the collective mode is entirely decoupled from all other ensemble modes due to our approximation of perfectly harmonic and homogeneous trapping. We allow for corrections to this simplified model by introducing a term H_{γ_m} representing coupling of the collective mode to a bath, which, in addition to including effects of mixing with other axial modes, might include effects of radial motion or light-induced effects beyond the optomechanical interaction included in H_{sys} . Further accounting for a coupling H_κ of the intracavity field to input field modes with energy H_{drive} , the full optomechanical Hamiltonian takes the form

$$H_{\text{tot}} = \hbar \left[\omega_0 + \delta\omega_N + \mathcal{G} X_0 (\hat{c}^\dagger + \hat{c}) \right] (\hat{a}^\dagger \hat{a} - \langle \hat{a}^\dagger \hat{a} \rangle) + \hbar \omega_t \hat{c}^\dagger \hat{c} + H_{\text{drive}} + H_\kappa + H_{\gamma_m}, \quad (3)$$

where we have subtracted an offset associated with the average intracavity light level and absorbed the associated force on the atoms into a redefinition of the trap centers ξ_i . In terms of the zero-point length $X_0 \equiv \sqrt{\hbar/(2M\omega_t)}$, the annihilation operator \hat{c} for mode \mathcal{C} has the usual definition such that $\hat{X} = X_0(\hat{c}^\dagger + \hat{c})$ and $\hat{P} = iM\omega_t X_0(\hat{c}^\dagger - \hat{c})$.

II. MECHANICAL MOTION AND TRANSMISSION FLUCTUATIONS

A. Equations of Motion

Equation 3 is the standard optomechanical Hamiltonian [2, 4] describing a cavity mode, with operator \hat{a} , whose frequency is shifted in proportion to the position $X_0(\hat{c}^\dagger + \hat{c})$ of a mechanical oscillator of frequency ω_t . The last three terms in Eq. 3 can be written out explicitly using the standard input-output formalism of quantum optics [5, 6]. One thereby derives equations of motion for \hat{a} and \hat{c} in terms of the optical input operators \hat{a}_{inj} , corresponding to the fields driving the cavity from its two ends labeled by $j \in 1, 2$, and a mechanical input operator \hat{c}_{in} corresponding to the thermal bath. We shall describe the evolution of the field operators in a rotating frame at the drive frequency ω_L , writing the intracavity field operator $\hat{a} = (\bar{a} + \hat{d})e^{-i\omega_L t}$ in terms of a c-number \bar{a} and a noise operator \hat{d} that accounts for small deviations from the classical value, and similarly letting $\hat{a}_{\text{inj}} = (\bar{a}_{\text{inj}} + \hat{d}_{\text{inj}})e^{-i\omega_L t}$. The classical field values are then in a steady-state relation

$$0 = (i\delta - \frac{\kappa}{2})\bar{a} - \sqrt{\frac{\kappa}{2}}(\bar{a}_{\text{in1}} + \bar{a}_{\text{in2}}), \quad (4)$$

while the deviations obey the linearized equations of motion [2]

$$\dot{\hat{d}} = (i\delta - \frac{\kappa}{2})\hat{d} + i\alpha(\hat{c} + \hat{c}^\dagger) - \sqrt{\frac{\kappa}{2}}(\hat{d}_{\text{in1}} + \hat{d}_{\text{in2}}), \quad (5)$$

$$\dot{\hat{c}} = (-i\omega_t - \frac{\gamma_m}{2})\hat{c} - \sqrt{\gamma_m}\hat{c}_{\text{in}} + i(\alpha^*\hat{d} + \alpha\hat{d}^\dagger), \quad (6)$$

where $\delta \equiv \omega_L - (\omega_0 + \delta\omega_N)$ represents the detuning of the drive field from cavity resonance for $X = 0$; and $\alpha \equiv -\mathcal{G}X_0\bar{a}$.

Defining the cavity response function $\chi_\kappa(\omega) = 1/[\kappa/2 - i(\omega + \delta)]$ and the mechanical response function $\chi_m(\omega) = 1/[\gamma_m/2 - i(\omega - \omega_t)]$, and rewriting the equations of motion in the Fourier domain, we have:

$$\tilde{d}/\chi_\kappa(\omega) = i\alpha[\tilde{c}(\omega) + \tilde{c}^\dagger(-\omega)] - \sqrt{\frac{\kappa}{2}}[\tilde{d}_{\text{in1}}(\omega) + \tilde{d}_{\text{in2}}(\omega)], \quad (7)$$

$$\tilde{c}/\chi_m(\omega) = i[\alpha^*\tilde{d}(\omega) + \alpha\tilde{d}^\dagger(-\omega)] - \sqrt{\gamma_m}\tilde{c}_{\text{in}}(\omega). \quad (8)$$

Here, $\tilde{\mathcal{O}}(\omega) \equiv \int_{-T/2}^{T/2} e^{i\omega t} \mathcal{O}(t) dt / \sqrt{T}$ denotes the windowed Fourier transform [6] of an operator \mathcal{O} , and in calculating spectra $\langle \tilde{\mathcal{O}}(\omega)\tilde{\mathcal{O}}(-\omega) \rangle$ (for Hermitian \mathcal{O}) we shall always implicitly take the limit $T \rightarrow \infty$ of a long window.

We can solve Eqs. 7-8 to obtain the dependence of $\tilde{X}(\omega)/X_0 = \tilde{c}(\omega) + \tilde{c}^\dagger(-\omega)$ on the optical input fluctuations $\hat{d}_{\text{in}} = (\hat{d}_{\text{in1}} + \hat{d}_{\text{in2}})/\sqrt{2}$ and the mechanical bath operator \hat{c}_{in} :

$$\tilde{c}(\omega) + \tilde{c}^\dagger(-\omega) = \frac{-\sqrt{\gamma_m}[\chi_m^{-1*}(-\omega)\tilde{c}_{\text{in}}(\omega) + \chi_m^{-1}(\omega)\tilde{c}_{\text{in}}^\dagger(-\omega)] - 2\sqrt{\kappa}\omega_t [\alpha^*\chi_\kappa(\omega)\tilde{d}_{\text{in}}(\omega) + \alpha\chi_\kappa^*(-\omega)\tilde{d}_{\text{in}}^\dagger(-\omega)]}{\mathcal{N}(\omega)}, \quad (9)$$

where

$$\mathcal{N}(\omega) = \chi_m^{-1}(\omega)\chi_m^{-1*}(-\omega) - 2i|\alpha|^2\omega_t\Pi(\omega) \quad (10)$$

and

$$\Pi(\omega) = \chi_\kappa(\omega) - \chi_\kappa^*(-\omega). \quad (11)$$

B. Input Field

We shall allow one end of the cavity to be driven by a laser at frequency ω_L that may have some phase noise. To account for this, we let $\hat{d}_{\text{in1}} = \hat{d}_{\text{in0}} + i\beta\bar{a}_{\text{in1}}$, where \hat{d}_{in0} represents quantum fluctuations, while $\beta(t) \ll 1$ is a real-valued stochastic variable representing the phase noise. At the other end of the cavity, we will admit only vacuum fluctuations (setting $\bar{a}_{\text{in2}} = 0$).

The quantum fluctuations ($j = 0, 2$) satisfy

$$\left\langle \tilde{d}_{\text{inj}}^\dagger(\omega) \tilde{d}_{\text{inj}}(\omega') \right\rangle = 0, \quad \left\langle \tilde{d}_{\text{inj}}(\omega) \tilde{d}_{\text{inj}}^\dagger(\omega') \right\rangle = \delta_T(\omega - \omega'), \quad (12)$$

where in the relevant limit $T \rightarrow \infty$, $\delta_T(0) = 1$ and $\delta_T(u) \rightarrow 0$ for $u \neq 0$. Phase noise modifies the corresponding relations for $\hat{d}_{\text{in}1}$,

$$\left\langle \tilde{d}_{\text{in}1}^\dagger(\omega) \tilde{d}_{\text{in}1}(\omega') \right\rangle = \left\langle \tilde{\beta}(-\omega) \tilde{\beta}(\omega') \right\rangle |\bar{a}_{\text{in}1}|^2, \quad \left\langle \tilde{d}_{\text{in}1}(\omega) \tilde{d}_{\text{in}1}^\dagger(\omega') \right\rangle = \delta_T(\omega - \omega') + \left\langle \tilde{\beta}(\omega) \tilde{\beta}(-\omega') \right\rangle |\bar{a}_{\text{in}1}|^2, \quad (13)$$

and adds correlations

$$\left\langle \tilde{d}_{\text{in}1}(\omega) \tilde{d}_{\text{in}1}(-\omega') \right\rangle = - \left\langle \tilde{\beta}(\omega) \tilde{\beta}(-\omega') \right\rangle \bar{a}_{\text{in}1}^2. \quad (14)$$

We will parameterize the laser noise by an effective linewidth $\gamma_L(\omega)$ given by the two-sided spectral density of frequency fluctuations, $\gamma_L(\omega) \equiv \omega^2 \left\langle \tilde{\beta}(\omega) \tilde{\beta}(-\omega) \right\rangle$. (For a laser with Lorentzian lineshape, γ_L is independent of frequency and represents the full width [7].)

C. Transmission Spectrum

We now proceed to calculate the two-sided spectrum $S_R^{(2)}(\omega)$ of cavity transmission fluctuations and relate this to the spectrum of the mechanical oscillator's motion. The rate R at which photons are transmitted from the cavity is given in terms of the output field operator $\hat{a}_{\text{out}} = \hat{a}_{\text{in}2} + \sqrt{\frac{\kappa}{2}} \hat{a}$ as $R = \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}$. The fluctuations of this rate about its mean value $\bar{R} = \bar{a}_{\text{out}}^2$ are given by $\hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}} - \bar{a}_{\text{out}}^2 \approx (\bar{a}_{\text{out}}^* \hat{d}_{\text{out}} + \bar{a} \hat{d}_{\text{out}}^\dagger)$, where $\hat{d}_{\text{out}} = \hat{a}_{\text{out}} - \bar{a}_{\text{out}}$ and we are working to lowest order in $\hat{d}_{\text{out}}/\bar{a}_{\text{out}}$. We assume, without loss of generality, that \bar{a} is real. Defining $\epsilon(\omega) \equiv \tilde{d}_{\text{out}}(\omega) + \tilde{d}_{\text{out}}^\dagger(-\omega)$, we then have $S_R^{(2)}(\omega)/\bar{R} = \langle \epsilon(\omega) \epsilon(-\omega) \rangle$. Using Eq. 7 to evaluate $\tilde{d}_{\text{out}} = \tilde{d}_{\text{in}2} + \sqrt{\kappa/2} \tilde{d}$, we find $\epsilon(\omega) = \epsilon_{\text{opt}}(\omega) + \epsilon_{\text{mech}}(\omega)$, where

$$\begin{aligned} \epsilon_{\text{opt}}(\omega) = & -(\kappa/2) \left[\chi_\kappa(\omega) \tilde{d}_{\text{in}1}(\omega) + \chi_\kappa^*(-\omega) \tilde{d}_{\text{in}1}^\dagger(-\omega) \right] \\ & + [1 - (\kappa/2) \chi_\kappa(\omega)] \tilde{d}_{\text{in}2}(\omega) + [1 - (\kappa/2) \chi_\kappa^*(-\omega)] \tilde{d}_{\text{in}2}^\dagger(-\omega) \end{aligned} \quad (15)$$

and

$$\epsilon_{\text{mech}}(\omega) = i\alpha \sqrt{\kappa/2} \Pi(\omega) [\tilde{c}(\omega) + \tilde{c}^\dagger(-\omega)]. \quad (16)$$

Here, ϵ_{opt} contains the intensity fluctuations due to photon shot noise or technical noise of the drive light, whereas ϵ_{mech} describes fluctuations in transmission due to atom-induced shifts of the cavity resonance.

Using Eqs. 15 and 16, we can express the fractional fluctuations in transmitted intensity as the sum of three terms describing, respectively, the intrinsic optical fluctuations; the motion-induced fluctuations; and the correlations between the first two:

$$S_R^{(2)}(\omega)/\bar{R}^2 = S_{\text{opt}}^{(2)}(\omega) + S_{\text{mech}}^{(2)}(\omega) + S_{\text{fb}}^{(2)}(\omega). \quad (17)$$

Here,

$$S_{\text{opt}}^{(2)}(\omega) = \langle \epsilon_{\text{opt}}(\omega) \epsilon_{\text{opt}}(-\omega) \rangle / \bar{R} = 1/\bar{R} + \gamma_L(\omega) |\Pi(\omega)|^2 \quad (18)$$

represents the optical fluctuations that would be present even in the absence of optomechanical coupling, namely photon shot noise and laser phase noise (converted into intensity noise by the cavity). $S_{\text{mech}}^{(2)}(\omega)$ is directly related to the spectrum $S_X^{(2)}(\omega) = \left\langle \tilde{X}(\omega) \tilde{X}(-\omega) \right\rangle$ of the mechanical motion (evaluated below in Eq. 21) by

$$S_{\text{mech}}^{(2)}(\omega) = \langle \epsilon_{\text{mech}}(\omega) \epsilon_{\text{mech}}(-\omega) \rangle / \bar{R} = \mathcal{G}^2 |\Pi(\omega)|^2 S_X^{(2)}(\omega). \quad (19)$$

Finally, the correlations between the mechanical motion and the optical noise are described by

$$\begin{aligned} S_{\text{fb}}^{(2)}(\omega) = & \langle \epsilon_{\text{opt}}(\omega) \epsilon_{\text{mech}}(-\omega) + \epsilon_{\text{mech}}(\omega) \epsilon_{\text{opt}}(-\omega) \rangle / \bar{R} \\ = & -4(\mathcal{G}X_0)^2 \omega_t \text{Im} \left[\frac{\Pi(\omega)}{\mathcal{N}(\omega)} \left(\chi_\kappa^*(\omega) + 2\gamma_L(\omega) |\Pi(\omega)|^2 \bar{R}/\kappa \right) \right]. \end{aligned} \quad (20)$$

D. Displacement Spectrum

We evaluate the two-sided spectrum of the mechanical oscillator's displacement $S_X^{(2)}(\omega) = \langle \tilde{X}(\omega)\tilde{X}(-\omega) \rangle$ using Eq. 9. Applying the simplest possible model for the bath, namely quantum white noise [8] with $\langle \tilde{c}_{\text{in}}^\dagger(\omega)\tilde{c}_{\text{in}}(\omega) \rangle \equiv \langle n_{\text{bath}} \rangle$, we obtain

$$S_X^{(2)}(\omega)/X_0^2 = \frac{\gamma_m \left[(\langle n_{\text{bath}} \rangle + 1) |\chi_m^{-1}(-\omega)|^2 + \langle n_{\text{bath}} \rangle |\chi_m^{-1}(\omega)|^2 \right] + 4\kappa |\omega_t \alpha \chi_\kappa(\omega)|^2 + 8[\gamma_L(\omega)\bar{R}/\kappa] |\omega_t \alpha \Pi(\omega)|^2}{|\mathcal{N}(\omega)|^2}. \quad (21)$$

The first pair of terms (in square brackets) describes motion arising from the oscillator's coupling to the bath. The middle term describes motion induced by photon shot noise of the probe light, while the last term describes motion induced by laser frequency noise.

Note that in the absence of optomechanical coupling ($\alpha = 0$), the oscillator spectrum $S_X^{(2)}(\omega)$ reduces to a pair of Lorentzians centered about $\pm\omega_t$

$$\left. \frac{S_X^{(2)}(\omega)}{X_0^2} \right|_{\alpha=0} = \gamma_m \left[(\langle n_{\text{bath}} \rangle + 1) |\chi_m(\omega)|^2 + \langle n_{\text{bath}} \rangle |\chi_m(-\omega)|^2 \right], \quad (22)$$

whose area is set by the bath temperature

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{S_X^{(2)}(\omega)}{X_0^2} = 2 \langle n_{\text{bath}} \rangle + 1. \quad (23)$$

E. Comparison with Measured Spectra

The spectra we measure are one-sided spectra, which we shall denote generically in terms of the two-sided spectra $S^{(2)}(\omega)$ by $S(\omega) \equiv S^{(2)}(\omega) + S^{(2)}(-\omega)$. The actual noise measured at the photodetector, normalized to the average photocurrent \bar{I} , is $S_I(\omega)/\bar{I}^2 = S_R(\omega)/\bar{R}^2 + \Sigma_{\text{det}}$. Here, Σ_{det} accounts for imperfect quantum efficiency $Q = 0.5(1)$, an excess noise factor $F = 4.5(5)$ of the avalanche photodiode, and dark (primarily Johnson) noise, and is given by

$$\Sigma_{\text{det}} = \frac{F - Q}{Q} \frac{2}{\bar{R}} + \frac{S_J}{(Q\bar{R})^2}, \quad (24)$$

where $S_J = 1.5(3) \times 10^9 / (\text{s}^2 \text{ Hz})$ expresses the measured dark noise in units of equivalent photon rate. The spectra in Fig. 4 are taken with a photon rate $\bar{R} = 1.2 \times 10^9/\text{s}$ at the output of the cavity, which yields $\Sigma_{\text{det}} = 1.6(1) \times 10^{-8}/\text{Hz}$.

We can now write the measured spectrum as $S_I(\omega)/\bar{I}^2 = S_{\text{mech}}(\omega) + S_{\text{bg}}(\omega)$, where

$$S_{\text{bg}}(\omega) = S_{\text{opt}}(\omega) + S_{\text{fb}}(\omega) + \Sigma_{\text{det}}. \quad (25)$$

From transmission noise $S_{\text{opt}}(\omega)$ measured at large photon rate in the absence of atoms, we have determined the effective laser linewidth to be $\gamma_L(\omega_t) = 2\pi \times 0.8(2)$ kHz at the trap frequency $\omega_t \approx 2\pi \times 500$ kHz (well within our 3 MHz lock bandwidth). At the photon rate $\bar{R} = 1.2 \times 10^9/\text{s}$ used in the spectra of Fig. 4, the associated phase-noise-induced intensity fluctuations are a factor of 1.7(5) below the photon shot noise level; correspondingly, they induce motion (included in our analysis) that is smaller than the zero-point fluctuations X_0 .

In fitting the measured spectra, we constrain F , Q , γ_L and \bar{R} ; we leave the dominant background noise contribution S_J free and obtain values consistent with the independently measured dark noise.

F. Determination of Collective Temperature

Subtracting the background level $S_{\text{bg}}(\omega)$ from the measured spectrum $S_I(\omega)/\bar{I}^2$ allows us to determine $S_X(\omega)$ from Eq. 21 and integrate it to find the occupation of the collective mode:

$$\langle n \rangle + 1/2 = \frac{1}{2} \int_0^\infty \frac{d\omega}{2\pi} S_X(\omega)/X_0^2. \quad (26)$$

To determine $\langle n \rangle$ in Fig. 2, we use not the spectrum itself but the fractional variance $\sigma_I^2 \equiv (I - \bar{I})^2 / \bar{I}^2$ of the measured photocurrent $I \propto R$ in a bandwidth $B \gg \omega_t$, which is related to the spectrum $S_I(\omega) / \bar{I}^2$ by

$$\sigma_I^2 = \int_0^B \frac{d\omega}{2\pi} S_I(\omega) / \bar{I}^2 \approx \int_0^\infty \frac{d\omega}{2\pi} S_{\text{mech}}(\omega) + \int_0^B \frac{d\omega}{2\pi} S_{\text{bg}}(\omega). \quad (27)$$

We make in Eq. 19 for $S_{\text{mech}}(\omega)$ the approximation $\Pi(\omega) \approx \Pi(\omega_t) = (2/\kappa)(\mathcal{L}_+ - \mathcal{L}_-)$, which yields

$$S_{\text{mech}}(\omega) \approx (2\mathcal{G}/\kappa)^2 |\mathcal{L}_+ - \mathcal{L}_-|^2 S_X(\omega). \quad (28)$$

Integrating Eq. 28 allows us to obtain $\int \frac{d\omega}{2\pi} S_X(\omega)$ from σ_I^2 using Eq. 27. The background noise term $\int_0^B \frac{d\omega}{2\pi} S_{\text{bg}}(\omega)$ on the right-hand side of Eq. 27 is independent of the occupation of the collective mode and is well approximated for the data in Fig. 2 by the variance $\sigma_{I,\text{eq}}^2$ measured in the long-time limit. In particular, we can find the change $\langle n \rangle - \langle n \rangle_{\text{eq}}$ in collective mode occupation between two different measurements $\sigma_I^2, \sigma_{I,\text{eq}}^2$ of the fractional transmission variance at fixed background noise by combining Eqs. 26-28:

$$\sigma_I^2 - \sigma_{I,\text{eq}}^2 = 8(\mathcal{G}X_0/\kappa)^2 |\mathcal{L}_+ - \mathcal{L}_-|^2 (\langle n \rangle - \langle n \rangle_{\text{eq}}). \quad (29)$$

The equilibrium occupation $\langle n \rangle_{\text{eq}} \lesssim 3$ of the collective mode \mathcal{C} is small compared to the values $\langle n \rangle$ plotted in Fig. 2, where mode \mathcal{C} is initially excited. Therefore neglecting $\langle n \rangle_{\text{eq}} \ll \langle n \rangle$, and reexpressing Eq. 29 in terms of transmission rate variances $\sigma^2 \equiv (R - \bar{R})^2 / \bar{R}^2 = \sigma_I^2 - B\Sigma_{\text{det}}$ and $\sigma_{\text{bg}}^2 \equiv \sigma_{I,\text{eq}}^2 - B\Sigma_{\text{det}}$, we obtain

$$\sigma^2 - \sigma_{\text{bg}}^2 = 8(\mathcal{G}X_0/\kappa)^2 |\mathcal{L}_+ - \mathcal{L}_-|^2 \langle n \rangle. \quad (30)$$

III. ATOM NUMBER CALIBRATION

We measure atom number [9] via the cavity shift $\delta\omega_N = NC\Omega$, where $C = N^{-1} \sum_{i=1}^N \sin^2(k\xi_i)$. Allowing for the small but non-zero radial cloud size $\sigma_r = 7(1) \mu\text{m} \ll w$, N represents an effective number of on-axis atoms. The cloud is long (≈ 1 mm) compared to the $5\text{-}\mu\text{m}$ beat length between trap and probe, so that $C = 1/2$ in the absence of probe light. Displacement of the atoms by the probe light reduces C by at most 12% in our experiments, and we account for this effect.

IV. THERMODYNAMIC TEMPERATURE

The thermodynamic axial temperature, given by the mean single-atom vibrational occupation number $\langle n_i \rangle$, is of interest for comparison with the bath temperature inferred from the fits in Fig. 4. We estimate $\langle n_i \rangle$, in an ensemble of $N = 1000(100)$ atoms, by ramping off the trap over $20 \mu\text{s} \gg 1/\omega_t$ while increasing the lattice depth of the probe to $U_p \approx 90\hbar\omega_t$. In the probe lattice, blue-detuned by $\Delta = +280$ MHz from atomic resonance, axially cold atoms localize at positions x'_i near the nodes. Before the cloud has time to expand radially, we determine the atom-probe coupling $C' = \langle \sin^2(kx'_i) \rangle$ via the cavity shift, normalized by the shift measured beforehand in the 851-nm lattice at $C = 1/2$. From $C' \approx (\langle n'_i \rangle + 1/2) \sqrt{E_r/U_p}$ we determine the mean vibrational level $\langle n'_i(0) \rangle = 6(2)$ in the final probe lattice. Note that this measurement provides only an upper bound on $\langle n_i \rangle \leq \langle n'_i \rangle$ if the transfer into the deep probe lattice is not entirely adiabatic.

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