Dissipative Preparation of Spin Squeezed Atomic Ensembles in a Steady State – Supplementary Material

Emanuele G. Dalla Torre[1],* Johannes Otterbach[1],* and Eugene Demler[1],Vladan Vuletic[2], Mikhail D. Lukin[1]
[1]*Physics Department, Harvard University, Cambridge 02138, MA, USA,*
[2]*Physics Department, Massachussetts Institute of Technology, Cambridge 02139, MA, USA*
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I. EXPERIMENTAL IMPLEMENTATIONS AND OPTICAL REPUMPING

In this section we investigate in detail the proposed experimental realization. Fig. 1 shows the relevant level-scheme of the ^{87}Rb $5S_{1/2}$ and $5P_{1/2}$ manifolds [1]. The cavity mode is a circularly polarized mode, which can be achieved using a running wave cavity with an optical element breaking the symmetry between the two degenerate polarization. Alternatively one can use a non-polarized cavity mode and apply a magentic field that lifts the Zeeman degeneracy. In this way, the relevant scheme can be made two-photon resonant, while keeping the complementary one (i.e. with opposite polarizations) two-photon off-resonant and thus strongly suppressed. If the control fields propagate along the cavity axis, the free spectral range of the cavity has to be choosen such that the cavity-mode and the control-fields are supported, or the power of the control-fields has to be made sufficiently large.

To derive the effective Hamiltonian (1) of the main text, we employ the Heisenberg-Langevin formalism [2]. Working far off single-photon resonance, i.e. $|\Delta|, |\Delta'| \gg \gamma$, we can neglect the spontaneous decay and consequently also the Langevin-noise forces. For simplicity of the calculation we assume that we can realize two independent Λ-schemes according to Fig. (1) and can neglect the respective second state. While this approximation is only true if the detunings from the respective excited state is smaller than the hyperfine-splitting in the excited state manifold, Hamiltonian (1) is valid for more general conditions, when Raman processes mediated by multiple virtual states, can be added up [3, 4]. The validity of this approximation will be further discussed below. With these approximations the equations of motion of

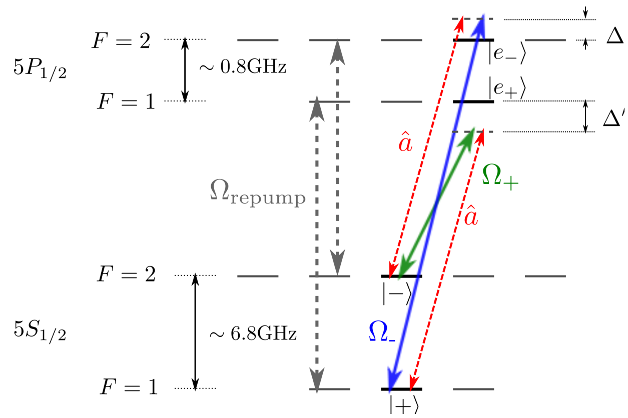

FIG. 1. Experimental level-scheme in ^{87}Rb using the clock states.

the atomic variables are given by

$$\frac{\partial}{\partial t}\hat{\sigma}_{+e_+} = -i\Delta'\hat{\sigma}_{+e_+} + i\Omega_-\hat{\sigma}_{+-} + ig\hat{\sigma}_{++}\hat{a} \tag{1}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{+e_-} = i\Delta\hat{\sigma}_{+e_-} + i\Omega_+\hat{\sigma}_{++} + ig\hat{\sigma}_{+-}\hat{a} \tag{2}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{-e_+} = -i\Delta'\hat{\sigma}_{-e_+} + i\Omega_-\hat{\sigma}_{--} + ig\hat{\sigma}_{-+}\hat{a} \tag{3}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{-e_-} = i\Delta\hat{\sigma}_{-e_-} + i\Omega_+\hat{\sigma}_{-+} + ig\hat{\sigma}_{--}\hat{a} \tag{4}$$

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\sigma}_{+-} = &-i\Omega_+\hat{\sigma}_{e_--} + i\Omega_-\hat{\sigma}_{+\mathrm{e}+} \\ &- ig\hat{\sigma}_{e_+-}\hat{a} + ig\hat{a}^\dagger\hat{\sigma}_{+e_-}\end{aligned} \tag{5}$$

Adiabatic elimination of all variables involving the excited states and subsequent insertion into Eq. (5) yields

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\sigma}_{+-} = &\, i\left(\frac{\Omega_+^2}{\Delta} + \frac{\Omega_-^2}{\Delta'}\right)\hat{\sigma}_{+-} \\ &+ ig\left(\frac{\Omega_+}{\Delta}\hat{a} - \frac{\Omega_-}{\Delta'}\hat{a}^\dagger\right)\hat{\sigma}_z - i\left(\frac{g^2}{\Delta} + \frac{g^2}{\Delta'}\right)\hat{a}^\dagger\hat{a}\hat{\sigma}_z\end{aligned} \tag{6}$$

We see that the off-resonant Raman coupling leads to an additional Stark-shift $\sim \left(\frac{\Omega_+^2}{\Delta} + \frac{\Omega_-^2}{\Delta'}\right)$ which, however, can be compensated by engineering a properly chosen two-photon detuning. The shift of the cavity resonance $\sim \left(\frac{g^2}{\Delta} + \frac{g^2}{\Delta'}\right)$ can be neglected as in the steady state the photon number in the cavity is zero. Neglecting these two frequency shifts, the above equation is equivalent to

*E.G. Dalla Torre and J. Otterbach equally contributed to this work the effective single particle Hamiltonian

$$h_{\text{eff}} = -ig\left(\frac{\Omega_+}{\Delta}\hat{a} - \frac{\Omega_-}{\Delta'}\hat{a}^\dagger\right)\hat{\sigma}_{-+} + \text{h.c.} \tag{7}$$

Defining the collective operators $\hat{S}_+ = \sum_j \hat{\sigma}_{+-}^{(j)}$, where the sum runs over all atoms, we find the final result

$$H_{\text{eff}} = \sum_j h_{\text{eff}}^{(j)} = \frac{g}{\Delta}\hat{a}^\dagger\left(\Omega_-\frac{\Delta}{\Delta'}\hat{S}_- - \Omega_+\hat{S}_+\right) + \text{h.c.} \tag{8}$$

Redefining $\Omega_-\frac{\Delta}{\Delta}' \to -\Omega_-$ results in Hamiltonian (1) of the main text. Hence, by adjusting relative phase and strength of the control fields the proposed Hamiltonian can be engineered.

In the derivation of Eqs. (1-5) we assumed the two Raman transitions involving Ω_+ and Ω_- to be independent. This approximation is valid only if both the detunings Δ and Δ' are smaller than the hyperfine-splitting δ_{HFS} in the excited state manifold. However, for the proposed realization in ^{87}Rb, the above condition cannot be fulfilled since the frequency of the cavity fixes the difference of the detunings to be equal to the difference of the hyperfine-splittings in the ground and excited state. Hnece we can at most choose one of the detunings to be smaller than δ_{HFS}. Nevertheless, as we will show now, in the system under consideration the coupling to the additional state interferes constructively giving rise to an effectively larger Raman scattering rate and thus rendering valid our approach.

We first consider the Raman transition involving Ω_- and denote the cavity dipole matrix-elements associated with the transitions $|-\rangle \to |e_-\rangle$ and $|-\rangle \to |e_+\rangle$, respectively, as g and g_2. Analogously, we define the Rabi frequency of the control field driving the transitions $|+\rangle \to |e_-\rangle$ and $|+\rangle \to |e_+\rangle$, respectively, as Ω_- and $\Omega_{-,2}$. Generalizing the above approach we find that the total Raman scattering rate is then given by

$$\Omega_{-,\text{tot}} = \frac{g\Omega_-}{\Delta} + \frac{g_2\Omega_{-,2}}{\Delta + \delta_{\text{HFS}}}\,. \tag{9}$$

The coupling matrix-elements can be expressed as $g = \wp_{-,e_-}\sqrt{\omega_c/2\hbar\epsilon_0 V}$ and $g_2 = \wp_{-,e_+}\sqrt{\omega_a/2\hbar\epsilon_0 V}$ where $\wp_{-,e_\pm}$ is the dipole matrix-element of the respective transition. For the ^{87}Rb D1 line these matrix-elements are given by $\wp_{-,e_-} = \sqrt{1/4}\langle J = 1/2||er||J' = 1/2\rangle$, $\wp_{-,e_+} = \sqrt{1/12}\langle J = 1/2||er||J' = 1/2\rangle$, with $\langle J = 1/2||er||J' = 1/2\rangle$ being the reduced matrix-element [1]. Analogously, the Rabi frequencies of the control fields are given by $\Omega_- = \wp_{+,e_-}E_-/\hbar$ and $\Omega_{-,2} = \wp_{+,e_+}E_-/\hbar$, where E_- is the electric field amplitude of the control field. The matrix-elements are given by $\wp_{+,e_-} = -\sqrt{1/4}\langle J = 1/2||er||J' = 1/2\rangle$, $\wp_{+,e_+} = -\sqrt{1/12}\langle J = 1/2||er||J' = 1/2\rangle$. Plugging these results into Eq. (9) we

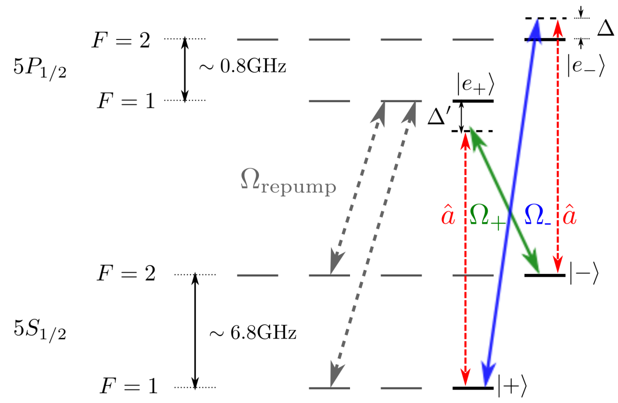

FIG. 2. Alternate experimental level-scheme using the stretched states of ^{87}Rb.

find

$$\Omega_{-,\text{tot}} = \left(\frac{1}{4\Delta} + \frac{1}{12(\Delta + \delta_{\text{HFS}})}\right) \times \frac{E_-}{\hbar}\sqrt{\frac{\omega_c}{2\hbar\epsilon_0 V}}\langle J = 1/2||er||J' = 1/2\rangle \tag{10}$$

Thus as long as Δ and $\Delta + \delta_{\text{HFS}}$ are of equal sign the two Raman rates constructively interfere giving rise to an effectively larger total Raman scattering rate. An analogous calculation for the transition involving the second control field Ω_+ yields a similar result. This shows, that as long as we choose the detunings such that we are always outside the hyper-fine structure, i.e. either blue or red detuned with respect to all hyperfine-states, we can always find a Raman scheme that gives rise to the effective Hamiltonian (1) of the main text.

As an alternate coupling scheme one could use the strechted states $|F = 1, m_F = 1\rangle$ and $|F = 2, m_F = 2\rangle$ in the $5S_{1/2}$ ground-state manifold as implementation of the spin-system (cf. Fig.2). Using linear polarized cavity fields along with circular polarization for the control fields results in the same effective Hamiltonian, once relative phase and strength of the control fields are chosen. However, this setup needs a different experimental geometry with the pump-beams entering the cavity from the side and the atoms trapped in a λ-lattice, to ensure that the spatial variations of the coupling matrix-elements have the same sign for all atoms.

Let us now address the question of spontaneous decay of the atoms into state outside the effective spin-system. This leads to two effects (i) a renormalization of the single atom cooperativity and (ii) to a slow decay of the number of atoms contributing to the spin-system. To see the first effect we note, that the presence of additional decay channels gives rise to a larger bandwidth of the excited states $|e_\pm\rangle$. Hence we have to replace the original decay rate γ by a decay-rate that also incorporates the decay rate γ_0 into other states, i.e. $\gamma \to \gamma + \gamma_0$, which, in turn, renormalizes the single-atom cooperativity to smaller values.

To circumvent the second effect, we propose to optically pump the states outside the spin-system back into the spin-states. For the realization involving a clock transition (cf. Fig 1), we note that the transition between the two clock-states $|\pm\rangle = |F = 1, 2, m_F = 0\rangle$ to the excited states $|F' = 1, 2, m_F = 0\rangle$ is forbidden. Hence any resonant linearly polarized light will bring the population back into the two clock states in analogy to optical pumping with linearly polarized light. Similar schemes can also be found for an experimental implementation using the strechted states of ^{87}Rb. The population in the external states is then given by the ratio between the decay-rates leading out of the spin-system and the repumping rate. In the limit of weak driving, i.e. $\Omega_{\text{repump}} \lesssim \gamma_0$, where Ω_{repump} is the Rabi frequency of the repump, and $\Omega_+ \lesssim |\Delta'|$, $\Omega_- \lesssim |\Delta|$, these rates are given by $\Gamma_{\text{out},\pm} = \gamma_0(\Omega_\pm/\Delta_\pm)^2$ and $\Gamma_{\text{repump}} = \Omega^2_{\text{repump}}/(\gamma_+ + \gamma_-)$. For large detuning, i.e. $\Delta^2 \gg \gamma_0(\gamma_+ + \gamma_-)$, this ratio can be made arbitrarily small, thus leaving only a small population in the external state.

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