SUPPLEMENTAL MATERIAL

Rate equation

The cooling force generated by the light scattered into the cavity mode is given by [S1]:

$$\mathbf{F_c} = \hbar \mathbf{k}\Gamma_{cav}\frac{4\delta_i \mathbf{k} \cdot \mathbf{v}}{(\kappa/2)^2 + \delta_{sc}^2},$$ (S1)

where

$$\delta_{sc} = \delta_i - \mathbf{k} \cdot \mathbf{v}$$ (S2)

is the detuning of the scattered light relative to the cavity resonant frequency, $\mathbf{k} = k\hat{x}$ is the wavevector of the incident light, and Γ_{cav} is the single-atom scattering rate into the cavity. The enhancement of the cavity scattering rate over free-space scattering rate is $\Gamma_{cav}/\Gamma_{sc} = \eta/(1 + (2\delta_i/\kappa)^2)$. Since the atoms are confined in the Lamb-Dicke regime along the cavity ($\hat{z}$) direction, only the force in the $\hat{x}$ direction is responsible for the cavity cooling process.

From the cooling force, we can calculate the rate at which thermal energy is removed from one atom:

$$\frac{d}{dt}W = (\mathbf{F_c} \cdot \mathbf{v})\Gamma_{cav} + 2E_{rec}\left[1 + \frac{\eta}{1 + (2\delta_i/\kappa)^2}\right]\Gamma_{sc} + H_{trap},$$ (S3)

where $W = \frac{3}{2}k_B T$ is the thermal energy of individual atoms. The rate equation can be written as:

$$\frac{d}{dt}\frac{3}{2}k_B T = -kv\frac{4\hbar kv}{\kappa}\frac{2\delta_i/\kappa}{[1 + (2\delta_i/\kappa)^2]^2}\eta\Gamma_{sc} +$$

$$2E_{rec}\left[1 + \frac{\eta}{1 + (2\delta_i/\kappa)^2}\right]\Gamma_{sc} + H_{trap}.$$ (S4)

By redefining $H_{trap} = \frac{3}{2}k_B h_{trap}$, we arrive at Eq. 1.

Photon-photon correlation function

For the light emitted from a large number of uncorrelated emitters, there is a simple relation between the first and second-order auto-correlation functions [S2]:

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2,$$ (S5)

where $g^{(1)}$ is defined as

$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle E^*(t)E(t)\rangle}.$$ (S6)

We consider N atoms with a Doppler width ω_D. The electric field of the scattered light field can be written as

$$\langle E^*(t)E(t+\tau)\rangle = E_0^2 \sum_{j=1}^{N} \frac{\exp i(\omega_0 + kv_{xj})\tau}{1 + \left[2(\delta_i + kv_{xj})/\kappa\right]^2}$$

$$= NE_0^2 \int_{-\infty}^{\infty} \frac{\exp i(\omega_0 + kv_x)\tau}{1 + \left[2(\delta_i + kv_x)/\kappa\right]^2}\exp\left(-\frac{k^2 v_x^2}{2\omega_D^2}\right)dv_x,$$ (S7)

where E_0 is the electric field amplitude scattered from an atom. The correlation function $g^{(2)}$ of the light scattered into the cavity mode is fit with Eqs. S5-S7 to obtain the Doppler width. The relation between Doppler width and temperature in one dimension ($\hat{x}$), $\omega_D = k\sqrt{\frac{k_B T}{M}}$, is used to extract the temperature of the atoms.

Heterodyne measurement

We interfere the emerging light from the cavity with a local oscillator 2 MHz detuned from the incident light on a 50/50 beam splitter. The output light from two ports of the beamsplitter is collected using SPCM-AQRH Single Photon Counting Modules from Excelitas Technologies, and its Fourier transform is calculated to extract the power spectrum of the light exiting the cavity. The average frequency shift $\delta\omega$ of the scattered light into the cavity relative to the incident light is a direct evidence of cavity cooling. Thermal energy is removed from individual atoms at a rate of $\Gamma_{cav}\hbar\delta\omega$, which equals the first term in the right hand side of Eq. S4. The linewidth of the spectrum is a product of the Doppler emission spectrum of the atoms $I(\omega) = I_0 \exp\left[-\frac{(\omega-\omega_0)^2}{2\omega_D^2}\right]$ and the cavity transmission profile $\mathcal{T}(\omega) = \frac{1}{1+[2(\omega-\omega_c)/\kappa]^2}$, where ω_0 is the frequency of the incident light, ω_c is the cavity resonance frequency, and ω_D is the Doppler width of the atomic ensemble. The temperature extracted from the emission spectrum of the cavity scattering light agrees well with that obtained from the $g^{(2)}$ function.

Cross thermalization

We rely on cross thermalization to realize 3D cooling with one incident beam. If the thermalization rate is not much higher than the cooling rate, Eq. 1 does not hold. The evolution of the temperature in the three directions is then determined by the following equations:

$$
\begin{aligned}
\frac{1}{2}k_B\frac{dT_x}{dt} &= -\frac{3}{2}k_B R_c\eta\Gamma_{sc}T_x + E_{rec}\Gamma_{sc}\left[\frac{7}{5} + \frac{\eta}{1+(2\delta_i/\kappa)^2}\right] + \frac{1}{2}k_B h_{trap} + \frac{R}{2}k_B(T_y - T_x) + \frac{R}{2}k_B(T_z - T_x) \\
\frac{1}{2}k_B\frac{dT_y}{dt} &= \frac{1}{5}E_{rec}\Gamma_{sc} + \frac{1}{2}k_B h_{trap} + \frac{R}{2}k_B(T_x - T_y) \\
\frac{1}{2}k_B\frac{dT_z}{dt} &= E_{rec}\Gamma_{sc}\left[\frac{2}{5} + \frac{\eta}{1+(2\delta_i/\kappa)^2}\right] + \frac{1}{2}k_B h_{trap} + \frac{R}{2}k_B(T_x - T_z),
\end{aligned}
\tag{S8}
$$

where R is the cross thermalization rate which is proportional to the elastic collision rate. We make the assumption that R is a constant during the cooling procedure.

First, we study the evolution of the temperature far from equilibrium. In this regime, the heating terms are negligible compared to the other terms. The equations for temperature in the $\hat{y}$ and $\hat{z}$ directions then reduce to be the same. We now denote $T_\perp = T_y = T_z$, and arrive at:

$$
\begin{aligned}
\frac{dT_x}{dt} &= -\big(3R_c\eta\Gamma_{sc} + 2R\big)T_x + 2RT_\perp \\
\frac{dT_y}{dt} &= R\big(T_x - T_\perp\big).
\end{aligned}
\tag{S9}
$$

The solution to Eq. S9 is a double exponential function for the temperature in both of the directions $T_{x,\perp} = a_{x,\perp}\exp\left(-\gamma_1 t\right) + b_{x,\perp}\exp\left(-\gamma_2 t\right)$. The rate constants are

$$
\gamma_{1,2} = 3R_c\eta\Gamma_{sc} + 2R - \frac{4R^2}{-(R + 3R_c\eta\Gamma_{sc}) \pm \sqrt{(R + 3R_c\eta\Gamma_{sc})^2 + 8R^2}},
\tag{S10}
$$

while the coefficients $a_{x,\perp}$ and $b_{x,\perp}$ are decided by the boundary conditions. The final temperature is obtained by adding the heating terms back, and it is the same as what we have from Eq. 1. In the limit of $R \gg 3R_c\eta\Gamma_{sc}$, the two rate constants become $R_c\eta\Gamma_{sc}$ (temperature drop) and $3R$ (cross thermalization), and the solution reduces to Eq. 1. With our experimental parameters ($R_c\eta\Gamma_{sc} \sim R$), the time constant for temperature drop deviates less than 45% from that provided in Eq. 1.

3D cooling with two cooling beams

As discussed in the main text, in the weak-confinement regime, one cooling beam along the $\hat{x}$ direction is generating a cooling force in the x-z plane, thereby realizing 2D cooling. Any second cooling beam not parallel to the $\hat{x}$ axis will thus generate 3D cooling. Here we present the data when we simultaneously and continuously send light from the $\hat{x}$
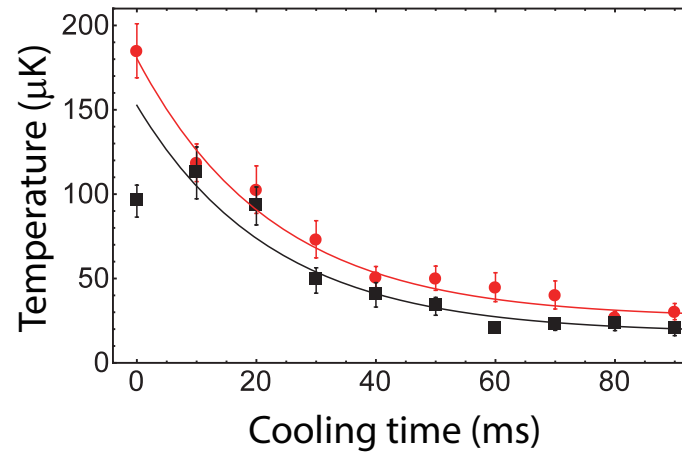
FIG. S1. Temperature in the $\hat{x}$ (red circles) and the $\hat{r}$ direction (black squares) as a function of cooling time when cooling beams are applied along both directions simultaneously.

and $\hat{r}$ direction and monitor the photon-photon correlation functions separately to extract the atomic temperatures in the two directions. The result shown in Fig.S1 shows a similar temperature reduction in both directions.

[S1] V. Vuletić, H. W. Chan, and A. T. Black, Phys. Rev. A **64**, 033405 (2001).
[S2] R. Loudon, *The quantum theory of light* (Oxford Science, 1983).