Fast Entanglement Distribution with Atomic Ensembles and Fluorescent Detection : Supplementary EPAPS Document

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In this supporting material, we provide details about how the rates and rate improvements shown in Fig. 3 of the paper were obtained.

In Fig. 3 of the paper we compare the rates achieved by our new scheme to those of previous proposals based on retrieval and single-photon detection. We consider schemes where no dark counts occur, and where the dominant error comes from multiexcitation events during entanglement generation. As explained in the main body of the paper, this is justified for our new protocol when purification by interrupted retrieval (PIR) is employed and the upper and lower bounds on the number of atoms are compatible. For the reference schemes, the assumption of no dark counts corresponds to an ideal situation. We also assume photon-number resolving detectors. Again, this corresponds to an ideal situation for the single-photon detections of the reference schemes, while for fluorescence detection, it is realistic to distinguish different occupation numbers for the relevant atomic levels.

I. SINGLE-RAIL

The single rail scheme considered in the paper is the original DLCZ scheme, proposed by Duan *et al.* in Ref. [1]. A generalised variant of this scheme was analysed in Ref. [5]. In that work, analytic expressions were derived for the rate and the quality of final entangled pairs in the DLCZ scheme under various conditions. The quality of entanglement was given in terms of a Bell parameter pertaining to a CHSH-type inequality. If we instead compute a more common figure of merit – the fidelity with respect to a maximally entangled Bell state – the expressions in the absence of dark counts become

$$R = \frac{2}{3} q\eta\, \eta_{spd}\, \frac{c}{L_0} e^{-L_0/L_{att}} (L/L_0)^{-\log_2 3} R' \qquad (1)$$

with

$$R' = e^{\frac{1}{\ln 2}\mathrm{Li}_2\left(\frac{\eta_{con}-1}{\eta_{con}}\sqrt{2}L/L_0\right) - \frac{1}{\ln 2}\mathrm{Li}_2\left(\frac{\eta_{con}-1}{\eta_{con}}\sqrt{2}\right)} \qquad (2)$$

for the rate, and

$$F = 1 - 6(L/L_0)^2(1-\eta_{con})\left(1 - \eta_{spd} e^{-L_0/L_{att}}\right) q\eta \qquad (3)$$

for the fidelity. Here η_{spd} denotes the single-photon detection efficiency, which applies to the detectors used for entanglement generation in both the new and reference schemes, and η_{con} denotes the detection efficiency during entanglement connection. Parameters L, L_0 and L_{att} denote the total length of the repeater, the segment length, and the attenuation length respectively, q is the probability for excitation, η is the fraction of light scattered forward, and Li_2 denotes the dilogarithm.

The connection detection efficiency η_{con} is different in the new and reference schemes. In the reference scheme, it is given by the product of the retrieval and single-photon detection efficiencies, while in the new scheme it equals the efficiency of fluorescence detection, taking into account the spin-wave loss incurred during PIR. For the reference scheme we thus have [4]

$$\eta_{con} = \left(1 - e^{-d/2}\left[\mathrm{I}_0(d/2) + \mathrm{I}_1(d/2)\right]\right)\eta_{spd}, \qquad (4)$$

where d is the optical depth and I_i denotes modified Bessel functions of the first kind. For the new scheme we have

$$\eta_{con} = (1 + \log(\eta)/d)\,\eta_{fl}, \qquad (5)$$

where η_{fl} is the fluorescence detection efficiency and η is the collection efficiency (see the main body of the paper).

By fixing the final fidelity, we can make use of (3) to eliminate $q\eta$ from (1). The resulting expression for the rate is then optimised over the number of primary segments to obtain the optimal rates in the new and reference schemes for each total distance. The rates obtained in this way and their ratio are shown in Fig. 3 in the paper, assuming $\eta_{spd} = 0.4$. In Fig. 1 below, we show the rates for a few values of η_{spd}.

II. DUAL-RAIL

The dual-rail scheme considered in the paper was proposed in Ref. [3], and a similar scheme was proposed in Ref. [2]. In these proposals, four different atomic ensembles are employed at each repeater node. In our scheme, there is only one ensemble per node and the atoms in use must therefore allow for four storage levels, rather than two as shown in Figure 1 of our paper for the single-rail scheme. The entanglement connection procedure is somewhat more complicated than in the single-rail case (see Fig. 1 of Ref. [3]). However, a 1-to-1 mapping from the linear optical transformations of Ref. [3] to atomic transformations in our scheme can be found, and fluorescence detection can again be substituted for retrieval and single-photon detection. In the dual-rail scheme, the rate at long distances scales as [3]

$$R = q\eta\,\eta_{spd}\,\frac{c}{L_0}e^{-L_0/L_{att}}(L/L_0)^{-\log_2 3}R' \tag{6}$$

with

$$R' = e^{-\log_2\left((2-\eta_{con})^4/\eta_{con}^2(3-2\eta_{con})\right)}. \tag{7}$$

As for the single-rail scheme, the excitation probability $q\eta$ can be eliminated by fixing the final fidelity, and the optimised rates for the dual-rail scheme can be obtained. In this case, $q\eta$ scales linearly with the distance, as opposed to quadratically for the single-rail scheme. The connection detection efficiencies are again given by (4) and (5) for the reference and the new scheme respectively. The rates for various values of η_{spd} are shown in Fig. 2. We note that the rates shown are obtained by direct numerical simulation along the lines of Ref. [3], rather than from the analytic expression above. This is slightly more accurate at short distances. For distances beyond a few hundred kilometres, the numerical and the analytic results are essentially identical.

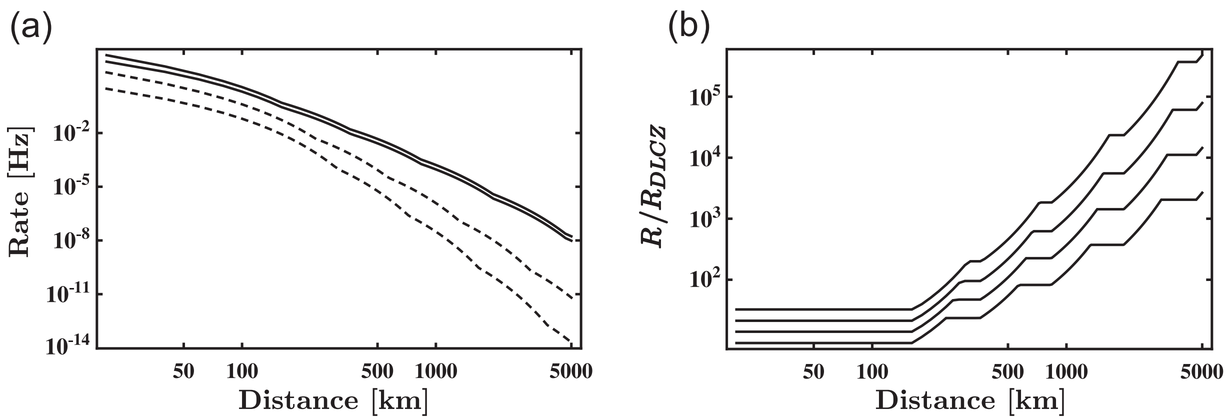

FIG. 1: **(a)** The rates in the new (solid) and DLCZ (dashed) single-rail schemes. The rates are shown for $\eta_{spd} = 0.4$ (upper curve) and 0.7 (lower curve). **(b)** Ratio of the rate in the new single-rail scheme to that of the DLCZ scheme for $\eta_{spd} = 0.4$, 0.5, 0.6, 0.7 (top to bottom). In both plots $\eta_{fl} = 0.95$, $\eta = 0.05$, $d = 100$, $L_{att} = 20$km.

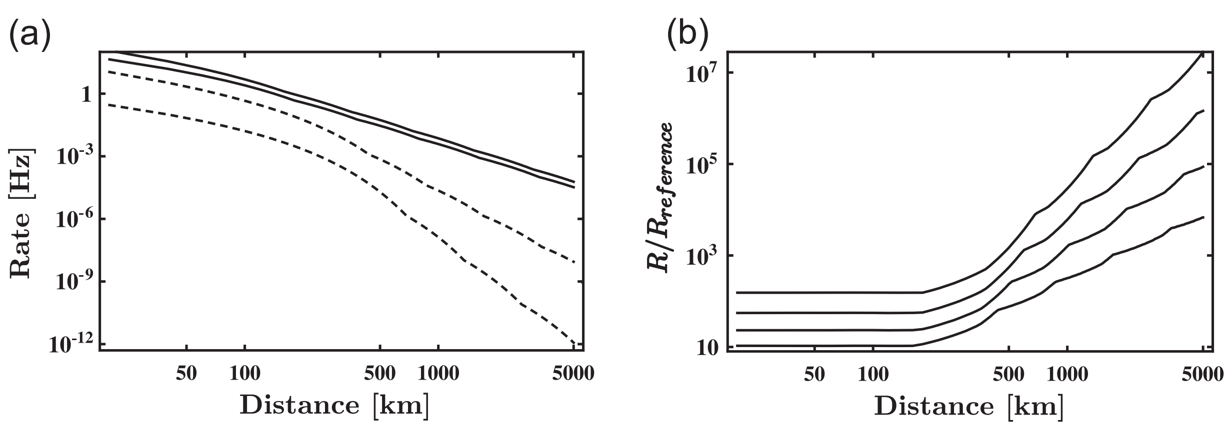

FIG. 2: **(a)** The rates in the new dual-rail scheme (solid) and the scheme of Ref. [3]. The rates are shown for $\eta_{spd} = 0.4$ (upper curve) and 0.7 (lower curve). **(b)** Ratio of the rate in the new dual-rail scheme to that of Ref. [3] for $\eta_{spd} = 0.4$, 0.5, 0.6, 0.7 (top to bottom). In both plots $\eta_{fl} = 0.95$, $\eta = 0.05$, $d = 100$, $L_{att} = 20$km.

III. ATTENUATION LEGTHS

As mentioned in the paper, the attenuation lengths pertaining to different repeater schemes depend on specific implementation since the wavelength of the transmitted light depends on which atoms and which atomic levels are chosen. We have verified that the advantage of our scheme persists even when the attenuation length in our scheme is significantly smaller than that of the the reference schemes. In Fig. 3 below, we show the improvement in rate when the attenuation length in our scheme is reduced by factors of 2 and 5 relative to that of the reference schemes.

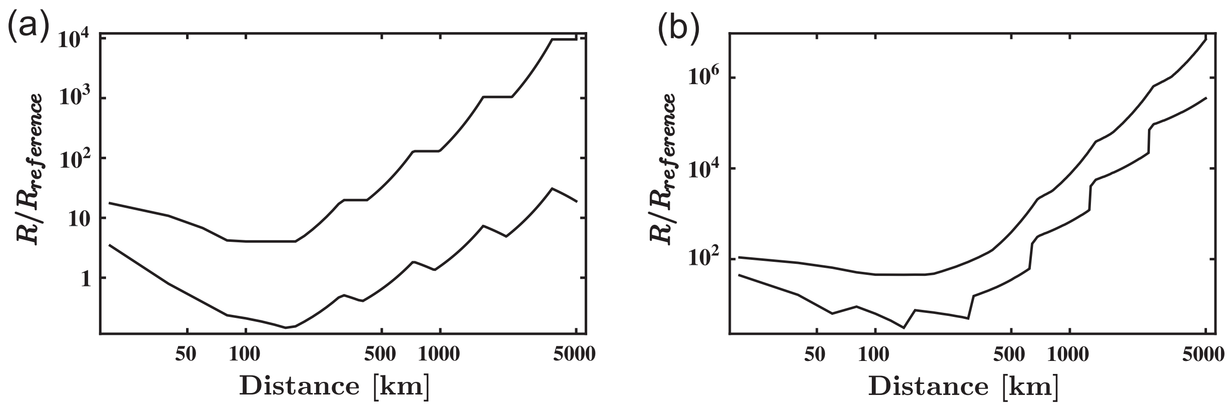

FIG. 3: **(a)** Ratio of the rate in the new single-rail scheme to that of the DLCZ scheme. **(b)** Ratio of the rate in the new dual-rail scheme to that of Ref. [3]. In both plots $\eta_{spd} = 0.4$, $\eta_{fl} = 0.95$, $\eta = 0.05$, $d = 100$ and $L_{att} = 10$km (top), 4km (bottom) in the new schemes and $L_{att} = 20$km in the reference schemes.

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