Supplemental Material for High-fidelity control and entanglement of Rydberg atom qubits

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I. STATE INITIALIZATION

We measure our state preparation fidelity by microwave spectroscopy. At our given magnetic field of 1.5 G, the transition frequencies from individual Zeeman sublevels of the ^{87}Rb $|F = 2\rangle$ ground state to the sublevels of the $|F = 1\rangle$ ground state are resolvable. We compare the microwave spectrum after coarse pumping into all sublevels of $|F = 2\rangle$ to the spectrum after pumping specifically into $|F = 2, m_F = -2\rangle$. In the latter measurement, peaks corresponding to initial population in magnetic sublevels other than $|m_F = -2\rangle$ are nearly completely eliminated. We bound their remaining size to produce our estimate that the initial state preparation has fidelity $> 99.5\%$. To reach this fidelity, we aligned the optical pumping laser to the magnetic field with reasonable care but without special fine tuning. However, we took great care to optimize the polarization purity to drive only σ^- transitions on the $|F = 2\rangle \rightarrow |F' = 2\rangle$ D_2 transition. We performed this optimization by directly measuring the atom signal while tuning the polarization.

II. CHARACTERIZING STATE DETECTION FIDELITY

The Rydberg pulse that couples the ground state $|g\rangle$ to the Rydberg state $|r\rangle$ is applied within a time window τ during which the optical tweezers are briefly turned off. Atoms that are left in $|g\rangle$ at the end of the pulse are recaptured by the optical tweezers, whereas those left in $|r\rangle$ are repelled by the tweezers and lost. We characterize this detection scheme by the ground state detection fidelity f_g (the probability for an atom left in $|g\rangle$ to be recaptured) and f_r (the probability for an atom left in $|r\rangle$ to be lost).

The ground state detection fidelity f_g for a given sequence in which the tweezers are turned off for time τ is estimated by leaving the Rydberg lasers off and measuring the loss probability due to just the trap-off period. For short trap-off times $\tau < 4$ μs, the loss probability is $< 1\%$ so that $f_g > 0.99$. Longer trap-off times reduce this fidelity.

The Rydberg detection fidelity f_r is characterized by the assumption that our single atom Rabi oscillation contrast is limited only by detection fidelity and finite coherence. This assumption is reasonable due to independent measurements which confirm our state preparation fidelity to be $> 99.5\%$. From our fitted Rabi oscillations, we extract the amplitude at time $t = 0$ to estimate the maximum possible loss signal which is typically $96(1)\%$. We associate the remaining 4% with detection error, such that the Rydberg detection fidelity is $f_r = 0.96(1)$. This is consistent with careful analysis of the loss mechanism described in [1].

III. RYDBERG LASER ALIGNMENT ONTO ATOMS

We ensure consistent, centered alignment of each Rydberg excitation laser by picking off a small portion of the beam as it is coming to a focus and redirecting it onto a reference CCD camera (see Figure 1b in the main text). We first identify the location on the CCD camera that corresponds to optimal alignment onto the atoms. This is done by systematically displacing the beam to several different positions, and at each position measuring the intensity on the atoms through a measurement of the light shift on the microwave transition from $|5S_{1/2}, F = 2, m_F = -2\rangle$ to $|5S_{1/2}, F = 1, m_F = -1\rangle$. We fit these measurements, along with their corresponding positions on the CCD, to extract the optimal position. This procedure takes 5 minutes. We then keep the beam aligned onto this position on the camera by feeding back to a computer controlled mirror located one focal length before the final focusing lens. This realignment takes < 5 seconds and is performed every 2 minutes. This alignment procedure is used on both beams.

IV. STABILIZING THE ELECTRIC FIELD ENVIRONMENT WITH AN ULTRAVIOLET LED

We observe drifts in the resonance frequency of the transition to the Rydberg 70S state on the scale of several MHz. We attribute these drifts to fluctuations in the electric field induced by charges on the surface of the glass cell which is 8 mm away from the atoms. We find experimentally that shining 365 nm ultraviolet light on the glass cell stabilizes the Rydberg resonance frequency. The most stable configuration is reached by applying the UV light for most of the experimental sequence; it is turned off only for a short window encompassing the Rydberg pulses. With this configuration, we can bound electric-field induced fluctuations on the Rydberg resonance to < 50 kHz.

V. SINGLE-ATOM PHASE GATE

We implement our single-atom phase gate with an independent 809 nm laser that is focused onto a single atom and shifts the energy of the atomic ground state $|g\rangle$ by $2\pi \times 5$ MHz. The laser is focused to a waist of 4 μm through a microscope objective positioned opposite to the primary microscope objective which produces the optical tweezers. The phase shift beam path is aligned counter-propagating to the path of the optical tweezer that traps the addressed atom.

The beam waist was chosen to be large enough such that the atomic light shift was insensitive to small temperature-induced position fluctuations of the atom. At the same time, the waist was chosen to be small enough to enable negligible crosstalk ($< 2\%$) between two atoms separated by 5.7 μm.

VI. NUMERICAL MODEL FOR SINGLE ATOMS

The numerical model is implemented using the Python package QuTiP [2]. It includes the following three effects:

1. A static but random Doppler shift in each iteration of the experiment. At 10 μK, in a counter-propagating beam configuration with wavelengths 420 nm and 1013 nm, the random Doppler shift follows a Gaussian distribution of width $2\pi \times 43.5$ kHz.

2. Off-resonant scattering from the intermediate state $|e\rangle = |6P_{3/2}, F = 3, m_F = -3\rangle$, to which both $|g\rangle$ and $|r\rangle$ are off-resonantly coupled by the 420 nm and 1013 nm lasers, respectively. The single-atom Rabi frequencies are $(\Omega_B, \Omega_R) \simeq 2\pi \times (54, 40)$ MHz, and the intermediate detuning is $\Delta \simeq 2\pi \times 540$ MHz.

   We experimentally probe the decay channels of $|e\rangle$ by preparing atoms in $|g\rangle$ and then applying only the 420 nm light for varying amounts of time. After the 420 nm light is applied at the usual power and detuning, we perform microwave spectroscopy between the $|F = 1\rangle$ and $|F = 2\rangle$ ground state manifolds to determine the final atomic populations. We estimate that population leaves the Zeeman sublevel $|g\rangle = |5S_{1/2}, F = 2, m_F = -2\rangle$ with a $1/e$ timescale of ~ 150 μs. We further estimate that population enters the $|F = 1\rangle$ manifold on a timescale of ~ 600 μs.

   The optical scattering rate induced by the blue laser at the known detuning and power, along with the known $6P_{3/2}$ lifetime of 115 ns, leads to an estimated scattering timescale of 45 μs, which is significantly faster than the timescale at which population leaves $|g\rangle$. The dominant decay channel from $|e\rangle$, then, is back into $|g\rangle$.

   We therefore make the simplification in the numerical model that scattering events from $|e\rangle$ return population to $|g\rangle$. This process is modeled by Lindblad operators $\sqrt{\gamma_B}|g\rangle\langle g|$ and $\sqrt{\gamma_R}|g\rangle\langle r|$, corresponding to scattering events from the ground state $|g\rangle$ or the Rydberg state $|r\rangle$. The simulated timescales are $\gamma_B = 1/(45\ \mu\text{s})$ and $\gamma_R = 1/(80\ \mu\text{s})$.

3. Finite lifetime of the Rydberg state $|r\rangle$. The total effective lifetime of 146 μs is composed of two decay channels: (1) blackbody stimulated transitions at 300 K to neighboring Rydberg states at a timescale of 230 μs and (2) radiative decay of the Rydberg state to low principal quantum number levels at a timescale of 410 μs [3].

   The simplified numerical model treats blackbody stimulated transitions as decay events into a new Rydberg state $|r'\rangle$ which is dark to the laser field. Additionally, the model accounts for radiative decay into a dark ground state $|g'\rangle$.

The numerical results are additionally rescaled to account for the independently calibrated detection fidelities.

VII. DEFINITION OF $|W\rangle$ STATE

For experiments with two interacting atoms (as described in Figures 3 and 4 in the main text), the two atoms are coupled homogeneously by the same laser field to the Rydberg state. Strictly speaking, the Hamiltonian is given by

$$\mathcal{H} = \frac{\hbar}{2} \sum_{i=1,2} \left[\Omega e^{ikx_i} |r_i\rangle\langle g_i| + h.c\right] + U|rr\rangle\langle rr|$$

The parameter Ω (taken to be real) is fixed by the laser intensities (which we assume here to be stable), and the parameter k is fixed by the combined wavevector of the two counter-propagating lasers. The parameters x_i describe the position of the two atoms along the array axis. In the Rydberg blockade regime where $U/\hbar \gg \Omega$, we project out the doubly excited state $|rr\rangle$ such that the only allowed levels are $|gg\rangle, |gr\rangle$, and $|rg\rangle$. We therefore rewrite the Hamiltonian as

$$\mathcal{H} = \frac{\hbar\Omega}{2} \left(e^{ikx_1}|rg\rangle\langle gg| + e^{ikx_2}|gr\rangle\langle gg| + h.c\right) \tag{1}$$

$$= \frac{\hbar\sqrt{2}\Omega}{2} \left(|W\rangle\langle gg| + |gg\rangle\langle W|\right) \tag{2}$$

where we have defined

$$|W\rangle = \frac{1}{\sqrt{2}} \left(e^{ikx_1}|rg\rangle + e^{ikx_2}|gr\rangle\right) \tag{3}$$

In the main text, we work in the rotating frame in which the excited state $|r\rangle$ incorporates these position-dependent phase factors. Strictly speaking, however, the definition of $|W\rangle$ depends on the position of the atoms.

At the finite atomic temperature of 10 μK, the atomic position along the array has a random Gaussian distribution of width ~ 200 nm. The relative phase $e^{ik(x_2-x_1)}$ may therefore reach appreciable values. We choose to not include these factors in our calculation of entangled state fidelity since they are fixed relative to the excitation laser system and do not emerge in measurements. Moreover, additional pulses that map Rydberg excitations down to other ground states may effectively erase these phase factors [4].

VIII. EXTRACTING OFF-DIAGONAL MATRIX ELEMENTS OF DENSITY OPERATOR

We consider an initial two-atom state ρ_0 that has the same measured populations as $|W\rangle = \frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)$ but unknown off-diagonal elements. Then ρ_0 can in general be expressed as

$$\rho_0 = \frac{1}{2}(|gr\rangle\langle gr| + |rg\rangle\langle rg|) + (\alpha|gr\rangle\langle rg| + h.c.) \tag{4}$$

with off-diagonal coherence $|\alpha| \leq 1/2$. We aim to measure α, and in doing so to measure the entanglement fidelity of the two-atom system.

We consider the following protocol to determine α. First, we apply a local phase shift operation that acts only on the left atom. This is achieved by a tightly focused laser which introduces a light shift on the ground state of the left atom. In the presence of this laser, $|gr\rangle$ is shifted by δ whereas $|rg\rangle$ is unshifted. After time t, the two states have accumulated dynamical phases $|gr\rangle \to e^{i\delta t}|gr\rangle$ and $|rg\rangle \to |rg\rangle$.

This operation transforms the density matrix from ρ_0 to ρ_ϕ where

$$\rho_\phi = \frac{1}{2} \left(|gr\rangle\langle gr| + |rg\rangle\langle rg|\right) + (\alpha e^{i\delta t}|gr\rangle\langle rg| + h.c.) \tag{5}$$

Note that if the state ρ_0 is a statistical mixture of $|gr\rangle$ and $|rg\rangle$ (that is, $\alpha = 0$), this phase shift operation does not change the density matrix.

We can rewrite ρ_ϕ from eq. (5) in the basis of the symmetric state $|W\rangle = \frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)$ and the orthogonal state $|D\rangle = \frac{1}{\sqrt{2}}(|gr\rangle - |rg\rangle)$:

$$\rho_\phi = \left(\frac{1}{2} + \alpha\cos(\delta t)\right)|W\rangle\langle W| + \left(\frac{1}{2} - \alpha\cos(\delta t)\right)|D\rangle\langle D| + \left[(-i\alpha\sin(\delta t))\,|W\rangle\langle D| + h.c.\right] \tag{6}$$

Finally, a global resonant π-pulse at the enhanced Rabi frequency $\sqrt{2}\Omega$ maps $|W\rangle \to |gg\rangle$. The probability to end in $|gg\rangle$ is therefore the probability to be in $|W\rangle$ after the phase shift operation. Therefore

$$P_{gg}(t) = \frac{1}{2} + \alpha\cos(\delta t) = 2\alpha\cos^2(\delta t/2) + \left(\frac{1}{2} - \alpha\right) \tag{7}$$

The amplitude of the oscillation of $P_{gg}(t)$ as a function of t therefore provides a direct measurement of α.

While this derivation holds only for an initial density matrix of the form given in eq. (4), a more general result can be found by considering the unitaries associated with the local phase shift, $Z_\phi^{(1)}$, and the π-pulse, X_π^W. In the basis $|gg\rangle, |gr\rangle, |rg\rangle, |rr\rangle$, the operators are given by:

$$Z_\phi^{(1)} = \begin{pmatrix} e^{i\delta t} & 0 & 0 & 0 \\ 0 & e^{i\delta t} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad X_\pi^W = \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{i}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

For an arbitrary initial density matrix ρ, the final density matrix is $\rho' = U\rho U^\dagger$ where U is the combined unitary $U = X_\pi^W Z_\phi^{(1)}$. The final probability to measure $|gg\rangle$ is then

$$P_{gg}(t) = \rho'_{gg,gg} = \frac{1}{2}\left(\rho_{gr,rg}e^{i\delta t} + \rho_{rg,gr}e^{-i\delta t}\right) + C \tag{9}$$

where the constant C is independent of the phase accumulation time t. Since $\rho_{gr,rg} = \rho^*_{rg,gr} = \alpha e^{i\theta}$ for some real amplitude α and angle θ, we have

$$P_{gg}(t) = \alpha\cos(\delta t + \theta) + C \tag{10}$$

We therefore see that for any initial density matrix, the amplitude of the oscillation of $P_{gg}(t)$ as a function of t gives a direct measurement of the off-diagonal coherence between $|gr\rangle$ and $|rg\rangle$.

IX. PROSPECTS FOR NEAR-TERM IMPROVED COHERENCE AND READOUT

We consider near-term achievable coherence based on a realistic improved laser system with 10 times the power on each transition while maintaining low phase noise. This will increase our single-photon Rabi frequencies to $\Omega_B = 2\pi \times 171$ MHz ($\Omega_R = 2\pi \times 126$ MHz). Balancing off-resonant scattering with the finite lifetime of the Rydberg state, we estimate that an optimal intermediate detuning in this configuration will be $\Delta = 2\pi \times 3.6$ GHz, resulting in a two-photon Rabi frequency of $\Omega = 2\pi \times 3$ MHz with a combined coherence time of 92 μs. In the absence of intensity fluctuations, this would result in $\Omega\tau = 276$, or equivalently a π-pulse fidelity of 0.998. In the same configuration, two atoms within the Rydberg blockade can be excited to the $|W\rangle$ state at the enhanced Rabi frequency in 118 ns with a fidelity of 0.999 (up to the breakdown of the blockade which can be reduced below 0.001 for reasonable separations between atoms).

For improving readout, we consider an approach involving the rapid mapping of Rydberg states to a secondary long-lived ground state. Without invoking a second Rydberg laser system, this mapping could be achieved by a coherent transfer from $|g\rangle$ to a second ground state $|g'\rangle$ followed by a π-pulse to map $|r\rangle$ to $|g\rangle$. The first pulse can be performed by a Raman laser system in < 0.5 μs (a π-pulse at 1 MHz, which has previously been achieved [5]), and the second pulse can be performed in 0.17 μs (a π-pulse at 3 MHz). Combined, the mapping can be performed with fidelity better than 0.99. Finally, state readout within the ground state manifold can be routinely accomplished with fidelity > 0.995 by pushing out $|F = 2\rangle$ atoms with cycling light on the $|F = 2\rangle \to |F' = 3\rangle$ transition.

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